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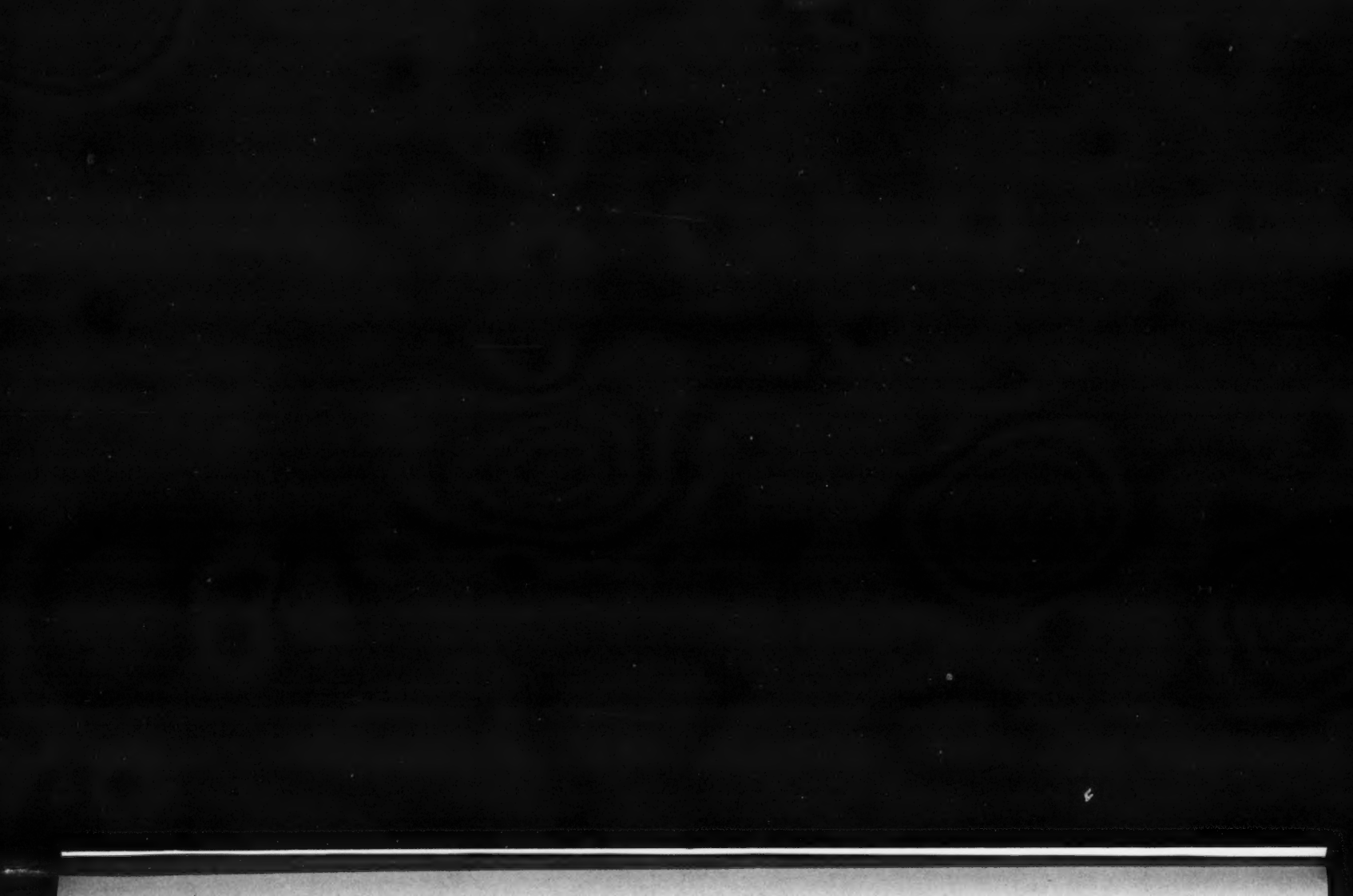

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On the Interpretation of Pd

Harold Jeffreys

(Received 1958 April 30)

Summary

Lehmann's observations of a large phase about 21°, lying a few seconds after the first arrival *Pr*, are discussed on two extreme hypotheses. On the first the distance varies as a quadratic function of sin *e*, so that $d\Delta/de$ vanishes at 21°, where there is a cusp in the time curve. The values of $dt/d\Delta$ for *Pr* and *Pd* are found to be such that there is no possible position for the nearer cusp. On the second, a cubic form for *t* is assumed up to 21° for *Pd*. *Pr* must enter between 17° and 18°. The *Pd* ray emerging at 21° must reach about 0·03 *R* below the Mohorovičić discontinuity. The times of *Pr* imply that at the transition there must be a considerable discontinuity of velocity gradient but only a small one of velocity. The nearer cusp would be near 15°, but as it lies less than a second after the first arrival it is probably undetectable.

1. Introduction

In the preparation of the Jeffreys–Bullen tables of 1939 I found that Lehmann's observations (1934), denoted by *Pd*, of a phase near 21° in the Azores earthquake of 1931 May 20, lay a few seconds too late to be attributable to a continuation of the *P* times at short distances. These data were therefore not used, and the *Pd* times were constructed, very tentatively, from data for Japanese deep focus earthquakes. The evidence against a single *P* phase in Japan has however disappeared, since the latest corrections (Jeffreys 1952 b, 1953) to the times of first arrival appear to account for the anomaly considered without further hypothesis.

On the other hand the revision of European *P* times (Jeffreys 1958) has shown that Lehmann's readings can now be fitted with $d^2t/d\Delta^2$ everywhere negative, and the possibility of a second phase needs re-examination. I have suggested (p. 159), on account of the large amplitudes, that the time curve may have a cusp near 21°, and the first problem is to construct a model with this property. Whether the velocity distribution is continuous or not, if there is a duplication of a phase there

must be a triplication, the time curve being of the general form shown in Figure 1. For a continuous distribution there are cusps at B and C and a node at A; for a discontinuous one BC corresponds to the reflected rays. OB and CD are curved downwards, BC upwards; the order $OABCAD$ is that of increasing angle of

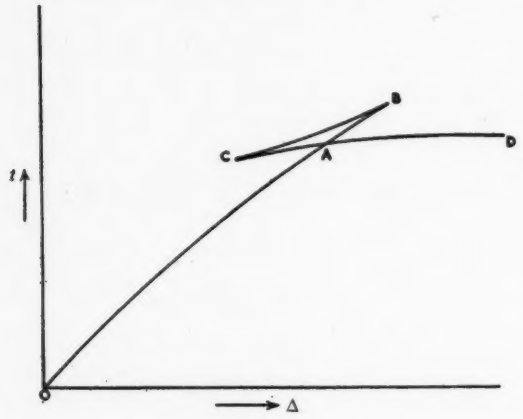

Fig. 1.

emergence e. The data are the times along OAD, with the extra point at B. It would be possible to fit them with a considerable range of extrapolations corresponding to BCA.

I consider two hypotheses. Apart from the slight complication due to the upper layers, on the first Δ is taken to be a quadratic in $\sin e$, such that $d\Delta/de$ vanishes at B. When the velocity gradient varies smoothly with depth but sufficiently rapidly to produce a receding branch, $d\Delta/de$ vanishes at the cusp. Bullen (1945) gives a detailed discussion. The adopted form is the simplest satisfying these conditions. On the second hypothesis the branch OB is taken to be cut off by a shadow; Δ is nearly proportional to e on OB. The solution will yield an estimate of the depth of the interface, and velocities at greater depths can be inferred by the usual method of "stripping" (Jeffreys 1952 a, Section 2.07).

2. First hypothesis

We consider rays between points at the top of the lower layer. The condition for a cusp is

$$d\Delta/de = 0,$$

and the simplest interpolation formula on OB is

$$\frac{d\Delta}{as} = a\frac{s_1 - s}{s_1},$$

where $s = \sin e$, $s_1 = \sin e_1$, $e_1 = e(B)$, and a is constant. This leads to expressions in finite terms

$$\Delta = \frac{a}{s_1}(s_1 s - \tfrac{1}{2}s^2),$$

$$t = \int \frac{R}{c_0} \cos e \, d\Delta = \frac{Ra}{c_0 s_1}\{s_1(\tfrac{1}{2}e + \tfrac{1}{4}\sin 2e) - \tfrac{1}{3}(1 - \cos^3 e)\}.$$

To order e^4

$$t - \frac{R}{c_0}\Delta = -\frac{Ra}{c_0 s_1}(\tfrac{1}{8}s^3 s_1 - \tfrac{1}{8}s^4).$$

For $s \ll s_1$,

$$t - \frac{R}{c_0}\Delta = -\frac{1}{6}\frac{R\Delta^3}{c_0 a^2},$$

and for $s = s_1$

$$t - \frac{R}{c_0}\Delta = -\frac{1}{3}\frac{R\Delta^3}{c_0 a^2}.$$

Since the last expression is only $-4{\cdot}6$ s out of nearly 5 min (Jeffreys 1958, Table 6 and top of p. 159) it would be unreasonable to expect any greater irregularity in the range of Δ considered.

A first approximation was used to estimate

$$p = dt/d\Delta$$

and to evaluate a correction for the upper layers (Jeffreys 1952 a, p. 72). I took a single surface layer giving a constant term $7{\cdot}7$ s at short distances, with the P velocity in it $5{\cdot}55$ km/s. This ignores the intermediate layer, but in view of the uncertainty of its properties and the fact that the whole effect of the upper layers is only a secondary consideration in the present problem there is no harm in replacing them by a single layer. The allowance is given in the following table (Table 1). The corresponding thickness of the upper layer is 28 km.

Table 1

Allowances for the upper layer

p s/deg	Allowance s
13·7	7·7
13	8·0
12	8·4
11	8·8
10	9·1
9	9·4
8	9·6
6	10·1
4	10·3
2	10·5
0	10·5

The times (Jeffreys 1958) after these allowances were made were as in Table 2.

Table 2

Corrected times over the Mohorovičić discontinuity

Δ	t	
deg	min	s
0	0	0·0
4	0	54·7
8	1	49·4
12	2	44·0
16	3	37·2
21	4	41·9

The next approximation to the solution gives

$$R/c_0 = 783{\cdot}8$$

$$s_1 = 0{\cdot}475, \qquad e_1 = 28^\circ\,22' = 0{\cdot}495,$$

$$a = 1{\cdot}54.$$

With these values t was calculated and the depth allowances were again added to give times over the outer surface. A slight change of R/c_0 was needed to remove negative residuals up to 16°, and the comparison is as in Table 3.

Table 3

Times over the outer surface

Δ	t (calc)		t (obs)		O–C	$dt/d\Delta$	
						Pd	*Pr*
deg	min	s	min	s	s	s/deg	s/deg
0	0	7·7				13·7	
4	1	2·4	1	2·4	0·0		
8		57·2		57·1	−0·1		
10	2	24·3	2	24·4	+0·1		
12		51·5		51·7	+0·2		
13	3	4·9	3	5·4	+0·5	13·6	
14		18·6		19·0	+0·4	13·5	
15		31·9		32·3	+0·4	13·4	
16		45·2		45·3	+0·1	13·4	
17		58·7		57·9	−0·8		
18	4	11·9	4	10·2	−1·7		12·1
19		25·0		22·2	−2·8		11·7
20		37·9		33·6	−4·3	12·9	11·2
21		50·8		44·5r	−6·3		
				50·3d	−0·5	11·9	
19		28·2					
16	3	53·2					

Values for 19° and 16° on the upper branch (*BC*) were calculated by extrapolating the formulae to $\sin e = 0{\cdot}5$ and 0·6. The values for first arrivals agree within the uncertainties up to 16°, but those from 17° onwards are too early for *Pd* and must be interpreted as *Pr*.

It would be doubtful whether the formulae could be applied to $e = 0{\cdot}6$, but $e = 0{\cdot}5$ needs only a small extrapolation, and if it was not permissible we might as well suppose *Pd* at 21° to be cut off by a shadow. Now *C* must be above the tangent to the upper branch at 19° (Jeffreys 1952, p. 49). It must also lie below the tangent to *Pr* at 18°. The respective values of $dt/d\Delta$ are 11·9 and 12·10 s/deg, that is, *Pr* is slightly the steeper, which is impossible. This might be due to observational uncertainty; but the tangent at 21° would give 4 min 16·0 s at 18°, 5·8 s later than *Pr*, and with tolerable estimates of uncertainty the two tangents could hardly be made to meet at any positive Δ. Δ for *C* would need to be smaller still. The tangent to *BC* at 19° would give 4 min 16·3 s at 18°, 6·1 s later than *Pr*, with the difference of slope in the wrong sense. It seems therefore that without additional complications it would be impossible to reconstruct the branches *BCA* on the hypothesis of continuous variation of velocity.

3. Second hypothesis

If the end of the *Pd* branch represents simply a shadow*, there is no reason against using an ordinary cubic formula up to that distance. The data are well fitted by the formula

$$t = 13{\cdot}71\Delta - 0{\cdot}612(\Delta/10°)^3$$

valid up to 17° and at 21°. To find an appropriate velocity distribution I use Bullen's r^{-k} law (1945). The depth reached by the ray that emerges at 21° is found to be 0·031 *R*. For constant velocity this depth would be 0·017 *R*, so that some increase of velocity with depth is still indicated. At 21° $dt/d\Delta$ is 12·90 s/deg. If the velocity is *c*, the time to order Δ^3 is

$$t = \frac{R}{c_0}\left(\Delta - \frac{(k+1)^2}{24}\Delta^3\right)$$

and $k+1$ is found to be 1·87.

Bullen does not give explicitly the method of allowing for focal depth with his law. If

$$c = c_0(R/r)^k$$

put

$$r^{k+1} = pc_0R^k \sec u.$$

Then the angular distance traversed is

$$[\theta] = \int \frac{p\,dr}{r(r^2/c^2 - p^2)^{\frac{1}{2}}} = \frac{1}{k+1}[u]$$

and

$$[t - p\theta] = \frac{p}{k+1}[\tan u - u].$$

These results apply to an origin at any depth. For an interface at depth 0·03 *R* I worked out the values of θ and *t* for the double passage down to and up from it and subtracted from the observed times, using values of *p* from 12·5 to 9·0 s/deg.

* This is not intended to imply that there is a shadow zone in the usual sense, but simply that the branch ends owing to a discontinuity of velocity of either sign. I cannot think of a suitable alternative word.

The resulting values of Δ and t below the discontinuity were interpolated, and are given as "obs." in Table 4.

Table 4

Times for a discontinuity of velocity at 0·03 R

Δ deg	t obs. sec	t calc. sec	O–C sec
0		0	
8	101·8	101·8	0·0
10	126·3	126·4	−0·1
12	150·4	150·2	+0·2
15	184·9	184·5	+0·4
16·5	200·9	210·0	−0·1
17·5	211·1	211·6	−0·5
19	226·8	226·9	−0·1
20·5	241·6	241·7	−0·1
21·5	251·3	251·1	+0·2

These correspond to times over the surface at distances 17° to 25°. They are represented by a cubic formula

$$t = 12{\cdot}901\Delta - 2{\cdot}65(\Delta/10)^3.$$

The observed times had been smoothed but represent about four independent estimates. The agreement is exceedingly good, and has been achieved more easily on this hypothesis than on any previous one. The coefficient of the cube term is of a reasonable magnitude, though it is four times as great as for the *Pd* layer and would imply a much more rapid increase of velocity with depth.

What is remarkable is that the coefficient of Δ is, to 0·01 s/deg, the same as $dt/d\Delta$ for *Pd* at 21°. Nothing is left for decrease of $dt/d\Delta$ along the reflected branch corresponding to *BC*. This would mean that the ray emerging as *Pr* at 21° is already just able to penetrate into the lower medium; that is, that there is no discontinuity of velocity, but only of the velocity gradient. It must be remembered that the coefficient of the cube term in the times of *Pd* has a standard error of about 1/8 of its amount, and $dt/d\Delta$ at its end may easily be as much as 13·1 s/deg, so that a small discontinuity of velocity is not excluded.

From these results the upper branches can be reconstructed as in Table 5.

Table 5

Travel times on the upper branch

p s/deg	Δ_0 deg	t_0 s	Δ_1 deg	t_1 s	Δ deg	t s	$t-t(Pd)$ s
12·9	14·5	198·2	1·0	12·9	15·5	211·1	+0·9
12·88	14·0	191·6	1·4	18·1	15·4	209·7	+0·8
12·86	13·5	185·1	2·0	25·8	15·5	210·9	+0·7
12·84	13·0	178·7	2·7	34·8	15·7	213·5	+0·6
12·82	12·6	173·6	3·3	42·4	15·9	216·0	+0·4
12·8	12·3	169·6	3·5	45·0	15·8	214·6	+0·4
12·7	10·9	151·9	5·0	64·1	15·9	216·0	+0·4
12·6	9·9	139·2	6·0	76·8	15·9	216·0	+0·4
12·5	9·2	130·5	7·0	89·4	16·2	219·0	+0·4

Suffix 0 indicates the layer between the two discontinuities, 1 that below the lower discontinuity, and Δ, t refer to distance and time between points on the Mohorovičić discontinuity.

The irregularities in Δ are of course only a result of end-figure errors. The change of p from the ray that reaches 21° (which in a closer computation would be about 12·93 s/deg) to 12·88 s/deg reduces Δ to 15°·4, and the slow increase of Δ from there for smaller p indicates that this must be very near the cusp C. The branch BC must be almost straight. Possibly there may be rays that penetrate the discontinuity and reach a distance a little greater than 21°; but I have not thought it worth while to investigate this further at present, as it would necessitate retaining a further figure in the computation. The interesting feature is that from 15°·4 to 16°·2 the branch CA lies less than a second after the first arrival. It has been pointed out that the hypothesis of a receding branch implies a nearer cusp, where there would be large amplitudes, and these have not been found. It now appears that the cusp lies so soon after the first arrival that in reading the seismogram the amplitudes would be referred to the first arrival, and there is no difficulty in understanding how it has failed to be detected.

4. Discussion

The whole of this paper depends on the reality of Lehmann's *Pd* readings in the Azores earthquake. She did not find the phase in the Iceland earthquake discussed in the same paper. I understand from private discussion that with one possible exception she has failed to find it in other earthquakes. It may therefore be a local peculiarity and in any case have to be added to the list of unexplained phenomena.

However, it has seemed worth while to test theoretically the hypothesis that *Pd* is a genuine phase. If it is, the hypothesis that it is due to a continuous velocity gradient appears untenable in its simplest form. The hypothesis that it is due to a shadow is consistent with the observed times up to 17°, but the times of *Pr* from 17° to 25° can be fitted with a discontinuity of velocity gradient, with no discontinuity of velocity or a very small one. The depth of the transition below the Mohorovičić discontinuity would be about 200 km. There is a concentration of energy at B, and large amplitudes could occur, which would not happen with an ordinary shadow due to discontinuity of velocity. There is still a receding branch, but it is due to rays that have penetrated below the interface.

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1958 *April* 29

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Dynamical Computation of the Equatorial Current System of the Pacific, with Special Application to the Equatorial Undercurrent

Koji Hidaka and Yutaka Nagata

(Received 1958 May 17)

Summary

The currents near the equator are treated by solving the dynamical equations for the steady-state involving terms of Coriolis force, pressure gradient, and horizontal and vertical mixing. The coefficients of horizontal and vertical mixing are estimated at 10^8 and 10^2 c.g.s. respectively and are constant everywhere. The distribution of mean wind-stress and mean pressure are computed from observed data. The result shows an eastward current below the surface near the equator, and it seems to give an explanation of the equatorial undercurrent.

1. Introduction

One of the difficulties of the geostrophic computation of ocean currents is that Bjerknes's method fails at or close to the equator because the Coriolis parameter vanishes there. In order to eliminate this singularity we must consider a more complete dynamical balance including, for example, the vertical and horizontal mixing. This kind of problem has been discussed by Sverdrup (1947), Reid (1948), Munk (1950), Weenink & Groen (1952), Yoshida, Mao & Horrer (1953), Cromwell (1953), Hidaka (1955), Tsuchiya (1955) and Yoshida (1955). Yoshida *et al.* considered a balance between the Coriolis force, pressure gradient, and the vertical and horizontal mixing, use being made of Reid's model for the vertical density distribution, and they calculated the current pattern at and around the equator from the distribution of wind-stress. Their result failed, however, to show the existence of the equatorial undercurrent which was discovered about 1952 by Hawaiian oceanographers (1953). On the other hand, we know the wind-stress on the sea surface and the pressure distribution from the observation. In an attempt to use the actually observed pressure distribution, Wyrtki (1956) computed the mass transport in the layer from the surface down to a depth of 200 m. His work gave a suggestion towards existence of the equatorial undercurrent. Recently Fofonoff & Montgomery (1955) explained the equatorial undercurrent in the light of the vorticity equation. In the present paper the result of our computation of velocity field in the meridional section across the equator will be given using the distribution of mean wind-stress and the mean pressure gradient computed from actually observed data, and then solving the steady-state dynamical equations.

2. Theory

Take the x-axis positive eastward, the y-axis northward and the z-axis vertically downward from the undisturbed level surface. Near the equator the east–west variation in velocity and pressure gradients is generally much smaller than the meridional variation, so we can neglect the term of $\partial^2/\partial x^2$ compared with the term in $\partial^2/\partial y^2$. Then the steady-state equations of motion are as follows:

$$\left\{\begin{array}{l} A_l\dfrac{\partial^2 u}{\partial y^2}+A_z\dfrac{\partial^2 u}{\partial z^2}+2\omega\rho\sin\phi\,.\,v=\dfrac{\partial p}{\partial x} \\ A_l\dfrac{\partial^2 v}{\partial y^2}+A_z\dfrac{\partial^2 v}{\partial z^2}-2\omega\rho\sin\phi\,.\,u=\dfrac{\partial p}{\partial y} \end{array}\right. \tag{1}$$

where u and v are the x-component and y-component of velocity respectively, p is the pressure, ρ the density of sea water, A_l the coefficient of horizontal mixing, A_z the coefficient of vertical mixing, ω the angular velocity of the Earth's rotation and ϕ the latitude. When we use the complex representation of velocity $\bar{w}=u+iv$, equations (1) become

$$A_l\frac{\partial^2\bar{w}}{\partial y^2}+A_z\frac{\partial^2\bar{w}}{\partial z^2}-i\,.\,2\omega\rho\sin\phi\,.\,\bar{w}=\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}. \tag{2}$$

For boundary conditions we take

$$\left\{\begin{array}{ll} z=0; & -A_z\dfrac{\partial\bar{w}}{\partial z}=\tau \\ z=1000\,\mathrm{m}; & \bar{w}=0 \end{array}\right. \tag{3}$$

where τ is the complex representation of wind-stress $\tau=\tau_x+i\tau_y$, and 1 000 m is an arbitrarily selected depth of no motion.

In equation (2) we replace the second derivative of $\bar{w}$ with respect to y with a finite difference approximation, or

$$\frac{\partial^2\bar{w}}{\partial y^2}\propto\frac{\bar{w}_{n-1}-2\bar{w}_n+\bar{w}_{n+1}}{(\Delta y)^2}$$

Then equation (2) becomes the following set of simultaneous equations;

$$\left\{\begin{array}{l} A_z\dfrac{d^2\bar{w}_1}{dz^2}+A_l\dfrac{\bar{w}_N-2\bar{w}_1+\bar{w}_2}{(\Delta y)^2}-i\,.\,2\omega\rho\sin\phi_1\,.\,\bar{w}_1=\left(\dfrac{\partial p}{\partial x}+i\dfrac{\partial p}{\partial y}\right)_1 \\ A_z\dfrac{d^2\bar{w}_2}{dz^2}+A_l\dfrac{\bar{w}_1-2\bar{w}_2+\bar{w}_3}{(\Delta y)^2}-i\,.\,2\omega\rho\sin\phi_2\,.\,\bar{w}_2=\left(\dfrac{\partial p}{\partial x}+i\dfrac{\partial p}{\partial y}\right)_2 \\ \cdots\cdots \\ A_z\dfrac{d^2\bar{w}_m}{dz^2}+A_l\dfrac{\bar{w}_{m-1}-2\bar{w}_m+\bar{w}_S}{(\Delta y)^2}-i\,.\,2\omega\rho\sin\phi_m\bar{w}_m=\left(\dfrac{\partial p}{\partial x}+i\dfrac{\partial p}{\partial y}\right)_m \end{array}\right. \tag{4}$$

where $\bar{w}_N$, $\bar{w}_S$ are the current velocities at the north and south boundaries of the domain under consideration. We shall assume that the flow there is geostrophic when the boundaries are far from the equator.

Multiplying both sides of the equations (4) by undetermined multipliers l_1, $l_2, \ldots, l_m$ respectively and adding together, we have

$$\begin{aligned} & A_z \frac{d^2}{dz^2}(l_1\bar{w}_1 + l_2\bar{w}_2 + \ldots\ldots + l_m\bar{w}_m) \\ & \quad + \frac{A_l}{(\Delta y)^2}\left\{\left(-2l_1 + l_2 - i\,.\,2\omega\rho\sin\phi_1\,.\,\frac{(\Delta y)^2}{A_l}l_1\right)\bar{w}_1\right. \\ & \quad + \left(l_1 - 2l_2 + l_3 - i\,.\,2\omega\rho\sin\phi_2\,.\,\frac{(\Delta y)^2}{A_l}l_2\right)\bar{w}_2 + \qquad (5) \\ & \ldots\ldots\ldots \\ & \quad \left. + \left(l_{m-1} - 2l_m - i\,.\,2\omega\rho\sin\phi_m\frac{(\Delta y)^2}{A_l}l_m\right)\bar{w}_m\right\} \\ & = (l_1\bar{w}_N + l_m\bar{w}_S)\,.\,\frac{A_l}{(\Delta y)^2} \\ & \quad + l_1\left(\frac{\partial p}{\partial x} + i\frac{\partial p}{\partial y}\right)_1 + l_2\left(\frac{\partial p}{\partial x} + i\frac{\partial p}{\partial y}\right)_2 + \ldots\ldots + l_m\left(\frac{\partial p}{\partial x} + i\frac{\partial p}{\partial y}\right)_m. \end{aligned}$$

Here we introduce the unknown ξ which satisfies the following equations:

$$\left\{\begin{array}{l} -\left(2 + i\,.\,2\omega\rho\dfrac{(\Delta y)^2}{A_l}\sin\phi_1\right)l_1 + l_2 = -(i\xi + 2)l_1 \\ l_1 - \left(2 + i\,.\,2\omega\rho\dfrac{(\Delta y)^2}{A_l}\sin\phi_2\right)l_2 + l_3 = -(i\xi + 2)l_2 \\ \ldots\ldots\ldots \\ l_{m-1} - \left(2 + i\,.\,2\omega\rho\dfrac{(\Delta y)^2}{A_l}\sin\phi_m\right)l = -(i\xi + 2)l_m \end{array}\right. \qquad (6)$$

Solving the characteristic equation (6) for ξ, we have m real roots ξ_j and corresponding sets of the multipliers $l_{1j}, l_{2j}, \ldots, l_{mj}$ $(j = 1, 2, \ldots, m)$.

Then equation (5) takes the following form;

$$\frac{d^2\bar{V}_j}{dz^2} - \lambda_j{}^2\bar{V}_j = F_j(z) \qquad (7)$$

where

$$\bar{V}_j = \sum_i l_{ij}\bar{w}_i \qquad (8)$$

$$\lambda_j{}^2 = \frac{A_l(2 + i\xi_j)}{A_z(\Delta y)^2} \qquad (9)$$

$$F_j(z) = -\frac{A_l}{A_z(\Delta y)^2}(l_{ij}\bar{w}_i + l_{mj}\bar{w}_S) + \frac{1}{A_z}\sum_i l_{ij}\left(\frac{\partial p}{\partial x} + i\frac{\partial p}{\partial y}\right)_i \qquad (10)$$

In the equations (7) and (10) the function $F(z)$ can be determined as a function of depth from observations of pressure. The boundary conditions (3) become then;

$$\begin{cases} z = 0 \quad ; \quad -A_z \dfrac{\partial \bar{V}_j}{\partial z} = \tau_j \\ z = 1\,000\,\text{m}; \quad \bar{V}_j = 0 \end{cases} \tag{11}$$

where

$$\tau_j = \sum_i l_{ij}\,\tau_i.$$

The solution of equation (7) becomes

$$\bar{V}_j = \frac{\tau_j}{\lambda_j A_z} \frac{\sinh \lambda_j (h-z)}{\cosh \lambda_j h} - \int_0^h F_j(s)\, k_j(z,s)\, ds \tag{12}$$

where

$$\begin{aligned} k_j(z,s) &= \frac{\sinh \lambda_j (h-s) \cosh \lambda_j z}{\lambda_j \cosh \lambda_j h} \qquad (s \geqslant z) \\ &= \frac{\sinh \lambda_j (h-z) \cosh \lambda_j s}{\lambda_j \cosh \lambda_j h} \qquad (s < z). \end{aligned} \tag{13}$$

Thus if we evaluate $\bar{V}_1, \bar{V}_2, \ldots, \bar{V}_m$ at each depth from the equation (12), we can get the current field by solving linear equations (8), or

$$\begin{cases} l_{11}\bar{w}_1 + l_{21}\bar{w}_2 + \ldots\ldots + l_{m1}\bar{w}_m = \bar{V}_1 \\ l_{12}\bar{w}_1 + l_{22}\bar{w}_2 + \ldots\ldots + l_{m2}\bar{w}_m = \bar{V}_2 \\ \ldots\ldots \\ l_{1m}\bar{w}_1 + l_{2m}\bar{w}_2 + \ldots\ldots + l_{mm}\bar{w}_m = \bar{V}_m \end{cases} \tag{8}'$$

3. Computation of the current velocity across a mid-Pacific meridional section

For practical computations, we take the north boundary at $\phi = 10°\,\text{N}$, and the south boundary at $\phi = 3°\,\text{S}$, and $\Delta y = 1°$. As to the coefficients A_l and A_z, values of 10^8 c.g.s. and 10^2 c.g.s. are chosen respectively.

For the wind-stress distribution, use has been made of SIO Oceanographic Report No. 14 (Figure 1). The y-component of pressure gradient $\partial p/\partial y$, has been computed by averaging the data of eight sections occupied between 158° W and 178° W (Figure 3). The average value of the x-component of pressure gradient between 3° N and 3° S is computed from the difference of the pressure at two sections which are about 3 000 km apart. We considered it to be independent of latitude because this factor gives only a small contribution to the calculated values (Figure 2).

The numerical results, u and v for each latitude, are tabulated in Tables 1 and 2. Values of u in meridional section are illustrated in Figure 4. The values of u, v and the contributions from pressure gradient and wind stress at surface are illustrated in Figure 5.

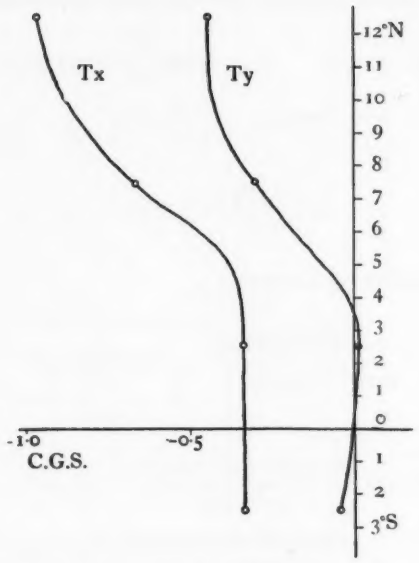

FIG. 1.—Annual mean distribution of wind-stresses between 160° W and 180° W from SIO Oceanographic Report No. 14.

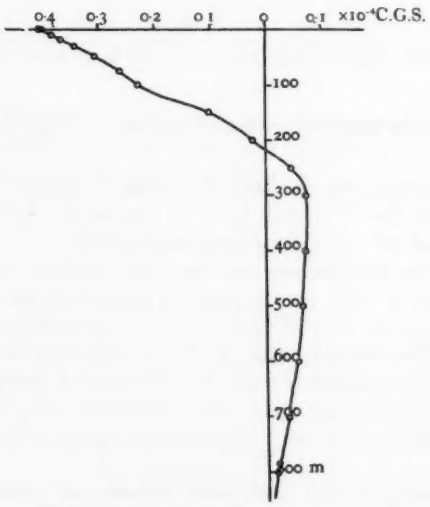

FIG. 2.—East–west component of pressure gradient $\partial p/\partial x$ averaged near the equator.

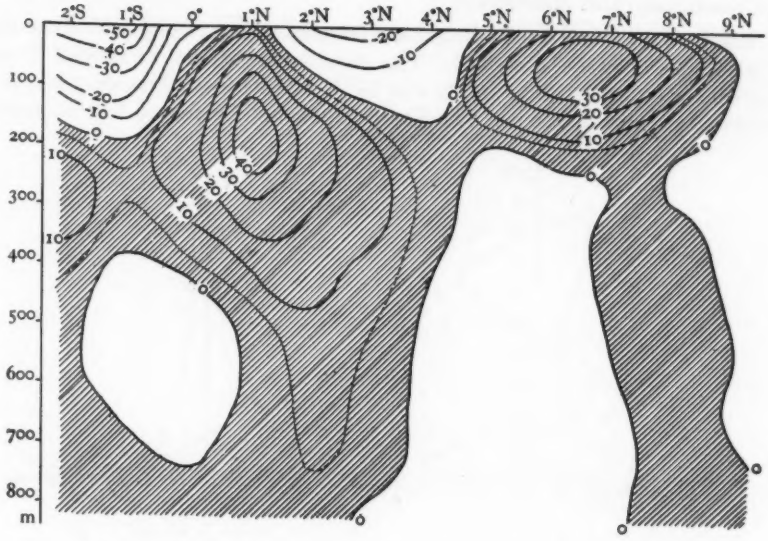

FIG. 3.—North–south component of pressure gradient $\partial p/\partial y$ computed by averaging the data of eight sections occupied acriss the equator between 158° W and 178° W ($\times 10^{-4}$ c.g.s.).

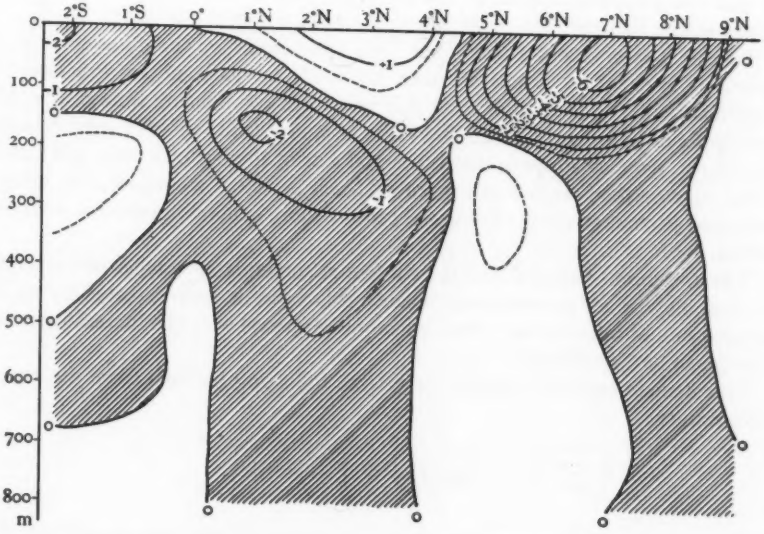

FIG. 4.—The derived distribution of east–west component of velocity in a meridional section across the equator (cm/s). Shaded areas show eastward flows.

4. Conclusion and discussion

Based on the above results the general features of the velocity field in the Equatorial Pacific may be described. The boundaries of the Equatorial Counter Current are located at about $4\frac{1}{2}°$ N and $8\frac{1}{2}°$ N and its lower boundary is approximately at a depth of 200 m. The South Equatorial Current is located south of

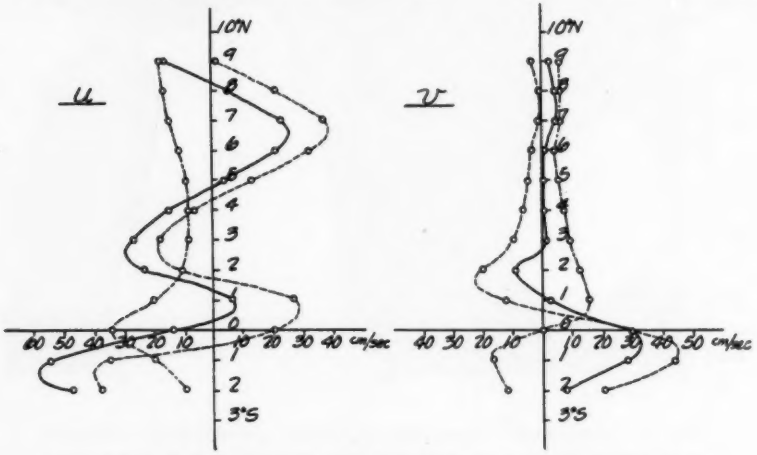

FIG. 5.—Profiles of u, v and the contributions from pressure (dashed line) and wind-stress(dot-dashed line) at surface.

$4\frac{1}{2}°$ N and its lower boundary is situated at about depth of 150 m. In Figure 4 we can see a typical equatorial undercurrent located below the surface approximately at the equator.

Its axis is seen at about 200 m at 1° N. The velocity of the undercurrent along its axis is about 1 kt. Since we take $\Delta y = 1°$, we have no information about phenomena of a scale smaller than this size. But considering that at $\phi = 0°$ the term $\partial p/\partial x$ most strongly affects the value of u, and while we only used an average value for it, we may suppose that the actual axis of the undercurrent will be closer to the equator than we see in Figure 4. Such an effect would tend to bring the computed current structure in Figure 4 into a more general agreement with observations.

The computed value of v at the surface at the equator reaches about 30 cm/s. But this may be an overestimate because this value is most affected by $\partial p/\partial y$ at the equator and because this value is likely to decrease with the increasing number of values of $\partial p/\partial y$ averaged.

To calculate vertical velocity, we get the following formula from the equation of continuity;

$$w(z) = -\int_0^z \frac{\partial v}{\partial y}\, dz$$

where we neglect the east–west component compared with the meridional. From our result, the pressure gradients are responsible for a tendency to sinking at the equator, while the drift components give a tendency of upwelling. But we cannot discuss the actual magnitudes of vertical motion at the equator because we did

Table 1

Computed values of u in meridional section cm/s.

z/φ	2°S	1°S	0°	1°N	2°N	3°N	4°N	5°N	6°N	7°N	8°N	9°N
0	−47·6	−54·8	−13·9	6·6	−23·2	−26·4	−15·0	3·6	20·9	23·0	4·8	−15·9
10	−44·4	−51·5	−10·6	9·6	−19·8	−23·1	−11·8	7·1	25·1	28·3	10·7	− 3·8
20	−41·6	−48·3	− 7·6	12·9	−16·7	−20·3	− 9·0	10·1	28·4	32·8	15·6	− 3·7
30	−38·9	−45·2	− 4·7	16·1	−14·0	−17·8	− 6·7	12·6	31·4	36·0	18·7	− 0·4
50	−34·8	−35·2	1·1	22·5	− 7·5	−13·7	− 3·4	17·4	35·3	38·8	20·6	2·0
75	−28·0	−32·1	5·8	30·7	− 0·2	− 9·4	− 1·0	18·7	36·6	37·4	18·9	1·3
100	−21·3	−24·7	10·3	37·8	7·2	− 5·4	− 0·1	18·2	34·2	32·4	14·5	− 0·1
150	− 6·0	−10·7	15·4	47·6	24·0	3·4	− 0·9	9·2	18·8	15·2	3·6	− 1·5
200	7·1	0·2	17·3	47·7	24·8	10·6	0·6	− 0·6	4·1	3·5	0·2	− 0·9
250	13·9	5·2	15·2	39·9	23·9	15·6	3·4	− 4·5	− 1·3	− 0·1	− 0·2	− 0·6
300	14·5	5·0	11·7	29·6	20·2	13·9	3·9	− 4·6	− 1·7	0·3	0·1	− 0·4
400	6·2	− 1·9	1·7	10·7	12·7	8·7	0·1	− 3·7	− 1·0	0·5	0·8	− 0·3
500	0·5	− 4·6	− 3·0	2·2	9·7	5·6	− 1·0	− 2·7	− 0·9	0·1	1·0	0·0
600	0·1	− 2·3	− 2·9	0·8	7·2	2·7	− 1·2	− 1·3	− 0·2	− 0·1	0·7	− 0·2
700	1·7	0·2	− 0·8	2·4	5·4	1·1	− 0·9	− 0·5	− 0·4	− 0·4	1·0	0·1
800	0·8	1·9	1·1	3·1	4·0	− 0·3	− 0·0	0·1	− 0·3	− 0·2	0·4	0·4
1 000	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0

Table 2

Computed values of v in meridional section in cm/s.

z/ϕ	2°S	1°S	0°	1°N	2°N	3°N	4°N	5°N	6°N	7°N	8°N	9°N
0	7·3	27·8	29·2	2·4	− 9·2	− 0·8	0·7	0·0	0·7	4·5	4·4	2·3
10	7·7	28·0	29·3	2·6	− 9·1	− 1·2	0·8	0·8	2·1	5·9	8·2	6·0
20	8·2	28·2	29·8	3·1	− 8·9	− 1·6	0·6	1·0	2·6	6·2	6·5	5·7
30	8·8	28·3	30·4	3·9	− 8·7	− 2·1	0·2	1·1	2·8	5·6	5·5	4·2
50	11·5	30·6	30·8	6·8	− 8·7	− 3·4	− 0·3	0·9	2·1	3·6	2·7	1·2
75	11·0	26·5	34·5	11·8	− 8·2	− 4·4	− 1·4	0·6	1·7	1·5	0·0	− 1·4
100	9·8	23·2	36·5	17·6	− 6·1	− 4·9	− 2·1	0·3	1·2	0·2	− 1·1	− 1·9
150	3·0	10·2	37·0	33·8	1·7	− 3·9	− 2·9	− 0·7	− 0·2	− 0·7	− 1·1	− 0·9
200	− 5·4	− 1·4	30·7	29·1	1·9	0·5	− 1·5	− 1·6	− 0·9	− 0·5	− 0·2	− 0·2
250	− 8·2	− 4·8	22·3	17·9	2·2	2·4	0·2	− 0·8	− 0·3	0·1	0·4	0·1
300	− 7·0	− 2·8	15·7	9·7	0·8	1·7	0·9	− 0·2	0·5	0·5	0·4	0·3
400	− 3·2	2·1	3·4	0·9	1·5	1·2	0·2	0·1	0·5	0·4	0·4	0·2
500	− 1·5	0·8	− 4·1	− 0·7	3·0	1·1	0·2	0·3	0·5	0·4	0·4	0·2
600	− 1·2	− 1·8	− 5·8	0·4	2·6	0·5	0·2	0·4	0·4	− 0·7	0·4	0·2
700	− 1·8	− 2·3	− 1·8	1·4	1·9	0·2	0·1	0·3	0·2	0·2	0·2	− 0·7
800	− 0·2	− 1·8	− 4·1	1·3	1·1	− 0·3	0·2	0·1	0·1	0·1	0·1	− 0·3
1 000	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0

not use the distribution of wind-stresses and the pressure gradients taken simultaneously.

In this paper computation of the velocity field in a meridional section across the equator has been made with a continuous and consistent system. We find that the *equatorial undercurrent* found by Hawaiian oceanographers about 1952 may be explained by solving the dynamical equations and inserting in them the wind stresses and pressure gradient computed from actual observations.

We cannot discuss further details with certainty because (1) a sufficient number of observations is not available to determine the distribution of mean values of $\partial p/\partial x$ and $\partial p/\partial y$ at or very close to the equator, (2) we cannot obtain the distribution of wind-stress and pressure gradient consistent with each other, (3) A_l and A_z have only assumed values constant everywhere, and we cannot test the sensitivity of the results to the magnitudes of A_l and A_z because the computations are very elaborate, and (4) the depth of the motionless layer is assumed at 1 000 m.

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1958 *May.*

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Progress Report on Long Period Seismographs

Hugo Benioff and Frank Press

(Received 1958 May 27)

Summary

Long period seismograph systems in operation in Pasadena are described. Extension of the group velocity curves for mantle Rayleigh waves and G-waves, the detection of these waves from earthquakes in the magnitude range $5\frac{3}{4}$–7, and the recording of unusual body waves with unsuspected long period components are among the results which have been achieved.

1. Instruments

For a number of years the research programme of the Seismological Laboratory of the California Institute of Technology has included the development of seismographs especially designed for extending the observed seismic spectrum to longer periods. The first of such instruments was an electromagnetic strain seismograph with galvanometric recording. This combination has a period response characteristic identical with that of a displacement transducer pendulum seismograph having a period and damping constant equal to the period and damping constant of the galvanometer. The original electromagnetic strain seismograph recorded with a galvanometer of 35 s period and thus was equivalent to a 35 s pendulum. A seismogram of the Japanese earthquake of 1933 March 2 made with this instrument clearly recorded mantle Rayleigh waves although they were not then recognized as such.

Some years later the galvanometer period was increased to 90 s and still later it was increased to 180 s (Benioff, Gutenberg & Richter 1954). This latter combination recorded mantle Rayleigh waves up to about 8 min period and other longer period wavelets in the Kamchatka earthquake of 1952 November 4. With the completion of the fused quartz extensometer at Isabella, California, a combination with a response equivalent to that of a pendulum of 10 min period was constructed. This employed a 10 min period galvanometer developed by Francis Lehner, coupled to the displacement transducer of the extensometer through a capacity-resistance differentiation network. Although the noise level in this combination is high owing to the response to surface temperature variations of the ground and to uncompensated barometric variations, it makes a satisfactory instrument for large earthquakes.

At Columbia University, Press and Ewing approached the problem of extension of the long wave portion of the spectrum by extensive modification of the Galitzin seismograph. Their first design in 1951 used a pendulum period of 15 s coupled to a galvanometer of 70 s. The vertical component instrument was provided with

a La Coste zero-length spring and in addition was compensated for barometric pressure variations. Both vertical and horizontal component units were designed to have negligible thermal response. In a later modification, carried out at Pasadena with Francis Lehner (Press, Ewing & Lehner 1958), the pendulum is operated at 30 s period with a Lehner galvanometer of 90 s period. Although the period bandwidth of this combination is less than that of the long period strain seismograph, it can be operated at much higher magnification owing in part to the restricted bandwidth. In addition it offers matched vertical and horizontal recording. The lack of vertical component response represents a serious limitation of the strain seismograph.

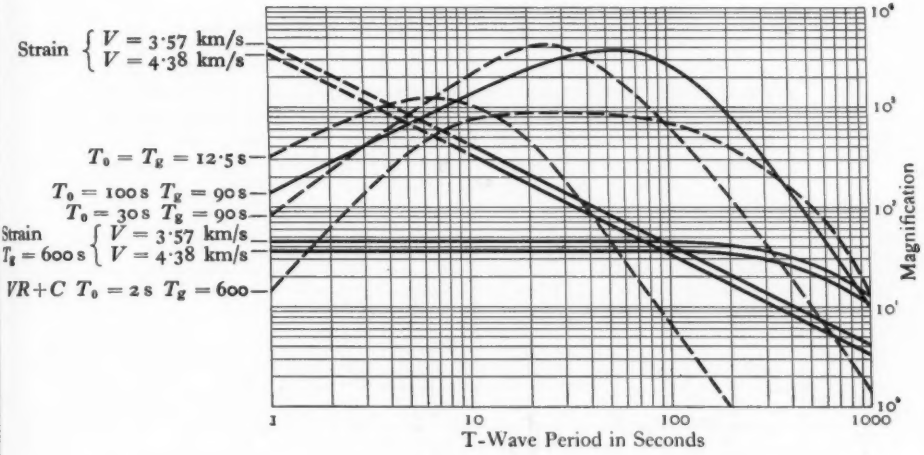

FIG. 1.—Response curves for long period seismograph systems. T_0—pendulum period; T_g—galvanometer period; $VR+C$—variable reluctance transducer with condenser shunt; V—phase velocity applicable to strain seismograph curve.

A simple modification making possible longer period pendulums was made by Ralph Gilman. It consists of a negative restoring force introduced into the pendulum system by means of a controlled DC current flowing through an auxiliary transducer. In this way he has increased the pendulum period to 100 s. A further modification of the Press-Ewing vertical instrument has extended the period of the vertical component seismometer to 80 s.

Another approach to long period recording made by Hugo Benioff uses a relatively short period reluctance pendulum seismometer ($T_0 = 2$ to 3 s) with a condenser shunted across the transducer coils and coupled to a 10 min period Lehner galvanometer.

Response curves of these instruments are presented in Figure 1. The different systems complement each other in bandwidth and magnification so that a wide portion of the spectrum is covered with relatively large dynamic range. One of the main advantages of long period seismographs such as these is the diminished response to waves with periods less than 20 s which obscure longer period phases on standard instruments.

2. Mantle Rayleigh waves

For wavelengths much larger than the thickness of the crust (periods greater than 75 s), Rayleigh wave dispersion is influenced primarily by the shear wave velocity variation with depth in the mantle and the vanishing rigidity in the core (Ewing & Press 1956). A further effect of the vertical velocity variation in the mantle is the production of maximal values in group velocity at 30 s periods for

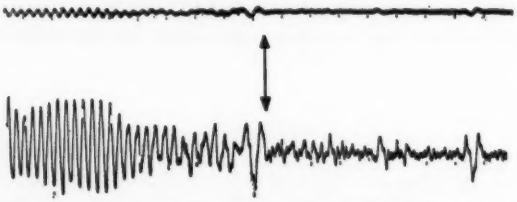

FIG. 2.—Mantle and crustal Rayleigh waves showing impulsive initiation corresponding to maximum value of group velocity. Loyalty Islands earthquake of 1956 November 26, magnitude 6¾. Upper trace Press-Ewing Z, $T_0 = 30$ s, $T_g = 90$ s; lower trace Benioff Z, $T_0 = 2$ s, $T_g = 180$ s, 100 mF shunt, $CRDX = 130\,000$ ohms.

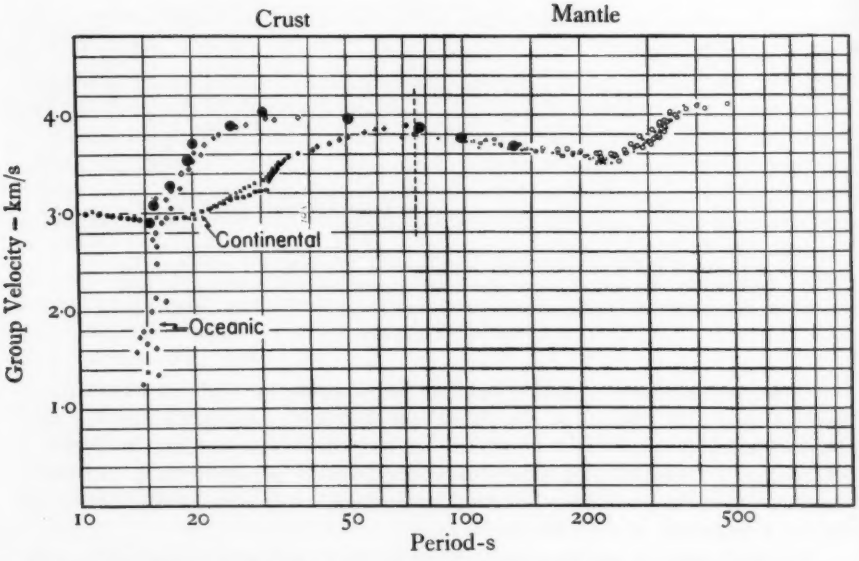

FIG. 3.—Mantle and crustal Rayleigh wave dispersion curve showing new data for maximum group velocity from earthquake of 1956 November 26. Oceanic data: Ewing & Press 1954. Continental data: Ewing (in press). Mantle data: Ewing & Press 1952. ● 1956 November 26, 23: 29: 41.

oceanic Rayleigh waves and 70 s periods for continental Rayleigh waves. Since large amplitudes are associated with stationary values of group velocity, oceanic Rayleigh waves are impulsively initiated on long period seismographs (Figure 2). Moreover the use of instruments with different responses permits separation of the regular and anomalous dispersion branches associated with the group velocity maximum. These dispersion data are plotted in Figure 3, together with results

from earlier studies. This represents the first time that crustal and mantle Rayleigh waves traversing the short arc could be analysed from the same earthquake. A gap in the dispersion curves for these waves was filled in this way.

There is a suggestion in the group velocity data of Figure 3 of a flattening of the curve for periods beyond 400 seconds. This has been interpreted as an effect of vanishing rigidity in the core. Additional experimental verification which places this trend in the group velocity curve on a more firm basis is shown in Figure 4.

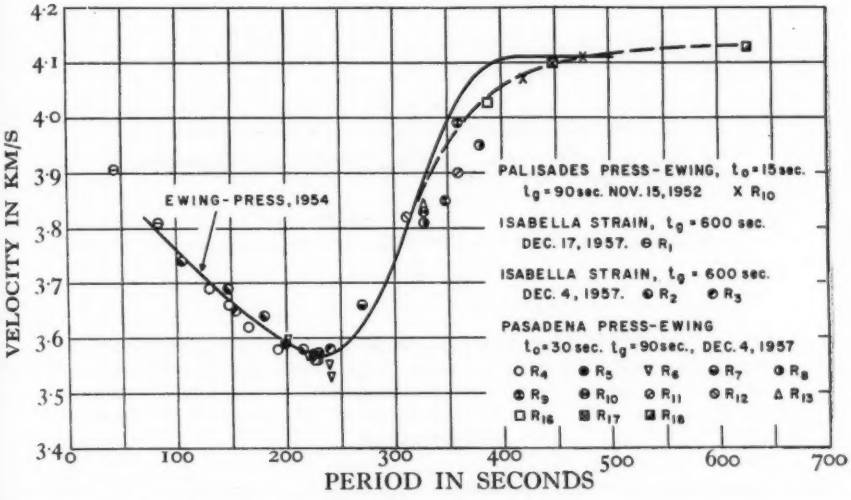

FIG. 4.—New data for mantle Rayleigh wave dispersion in the period range $T > 300$ s. Dashed curve represents modification.

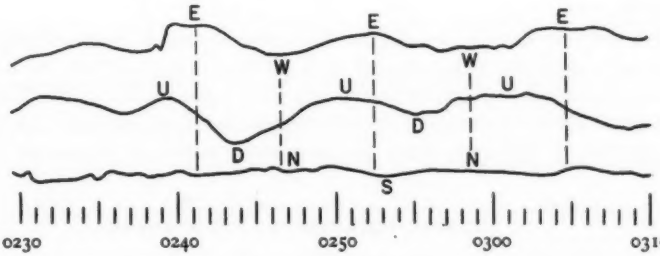

FIG. 5.—Three component seismograms $T_0 = 30$ s, $T_g = 90$ s showing mantle Rayleigh waves of order R_{18} from the Mongolian earthquake of 1957 December 4, magnitude 7·9.

Data for mantle Rayleigh waves of order R_{16}, R_{17}, R_{18} from the Mongolian earthquake 1957 December 4 were used. The seismograms for R_{18} are shown in Figure 5. This group travelled a distance of 349 600 km. The proper retrograde elliptical orbital motion for these waves, demonstrated by comparison of the E–W and vertical component traces, served to distinguish the signal from the background.

Previously, mantle Rayleigh waves were thought to be generated only by the greatest earthquakes. It is not uncommon with the long period instruments to

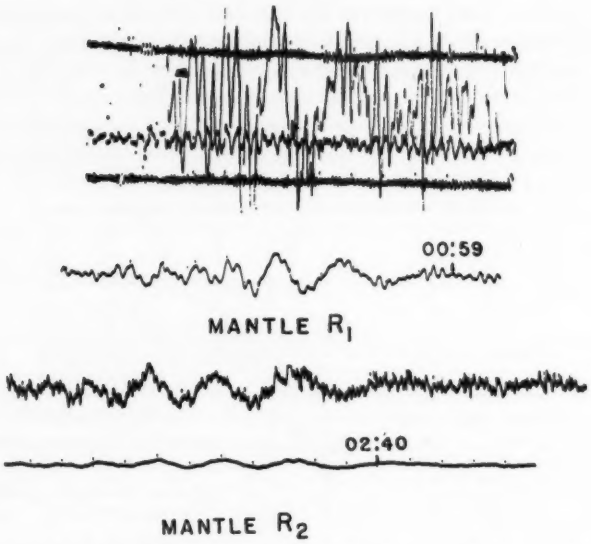

FIG. 6.—Mantle Rayleigh waves R_1 and R_2 corresponding to anomalous dispersion branch of group velocity curve. Tonga Island earthquake of 1956 December 27, h = 300 km, magnitude 7–7¼; Upper traces Z, T_0 = 30 s, T_g = 90 s; lower traces Z, T_0 = 2 s, T_g = 180 s, condenser shunt.

record these oscillations from teleseisms with magnitude as small as 6¾ (Figures 2 and 6). Note the greater suppression of shorter period crustal Rayleigh waves in the instrument T_0 = 2 s, T_g = 180 s.

3. G-Waves

G-waves are horizontally polarized shear waves in which the principal components propagate with the velocity 4·38 km/s (Benioff 1955). They are related to Love waves in that the velocity of the long period initial waves of the Love wave train exhibit the same velocity. It has been thought that *G* propagation is controlled by a flat portion in the Love wave group velocity curve as shown in Figure 7

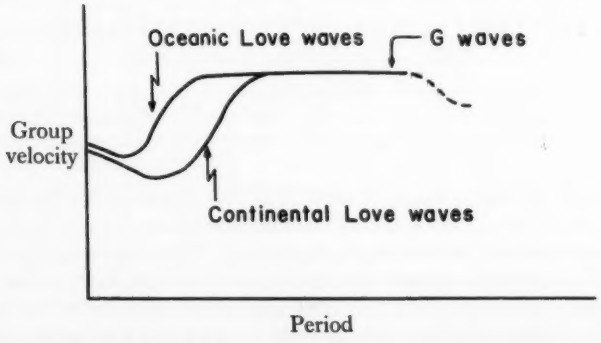

FIG. 7.—Schematic Love wave—*G*-wave dispersion curve.

by the heavy line. This would account for its impulsive initiation, and its efficient propagation. The different extent of the flat portion for the oceanic and continental curve would explain the observation that *G*-waves with oceanic paths occur typically as pulses (Figure 9) in contrast to the train of shorter period Love waves which develops from *G* after passage across a continent.

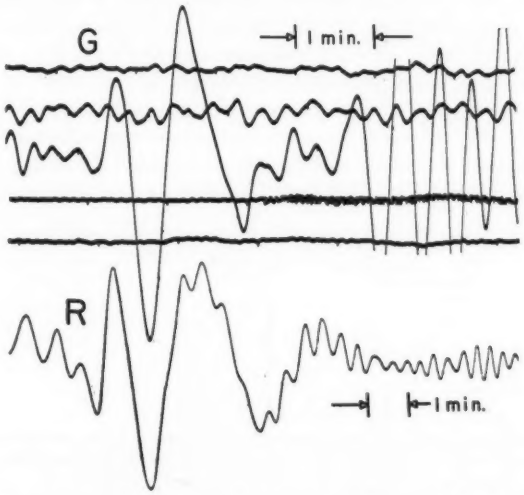

FIG. 8. *G*-wave anomalous dispersion corresponding to dashed portion of Figure 7. Mantle Rayleigh wave is shown for comparison.
G: Pasadena E–W $T_0 = 100$ s, $T_g = 90$ s 1958 April 13; 53° N, 161° E, $\Delta = 59°$, $M = 6\frac{1}{2}$.
R: Isabella Strain $T_g = 600$ s
1957 December 4; 45½° N, 99½° E, $\Delta = 94°$, $M = 7{\cdot}9$.

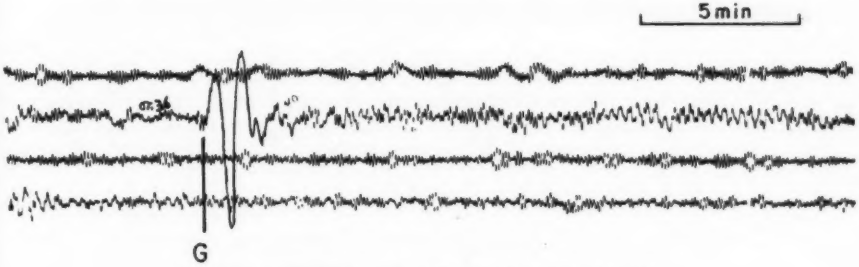

FIG. 9.—*G*-wave from small magnitude earthquake.
N–S, $T_0 = 30$ s, $T_g = 90$ s.
1956 September 24. 07: 02: 13, Fiji Islands, $\Delta \sim 85°$, $M \sim 5\frac{3}{4}$.

That the *G*-wave dispersion curve is a true maximum with a decrease in velocity for periods greater than about 2 min is shown by the seismogram in Figure 8, made with the Gilman E–W seismograph. A long period tail is shown for this *G*-wave with a character of striking similarity to the mantle Rayleigh waves also shown. It may be that this long period tail has been detected with the new instrument because the higher overall sensitivity permits use of *G* phases from earthquakes of smaller magnitude, hence of different spectrum from *G* generated by

larger earthquakes. The *G*-wave velocity is significantly less than the shear velocity of 4·7 km/s observed in the upper mantle. This may be the result of a low velocity layer in the mantle or simply a consequence of the occurrence of a maximum in group velocity due to the general velocity increase with depth in the mantle.

G previously was recorded for earthquakes of large magnitude. This was supposed to be a manifestation of the increase, with magnitude, of long period components in the spectrum of waves radiated from the source. *G*-wave recordings have been obtained from earthquakes with magnitude as small as $5\frac{3}{4}$ (Figure 9) making it more likely that the earlier correlation with magnitude was partly a matter of insufficient instrumental response to *G*-wave periods. That *G*-waves are efficiently excited by small earthquakes raises the interesting possibility that small, slow movements along faults with great extent may be involved.

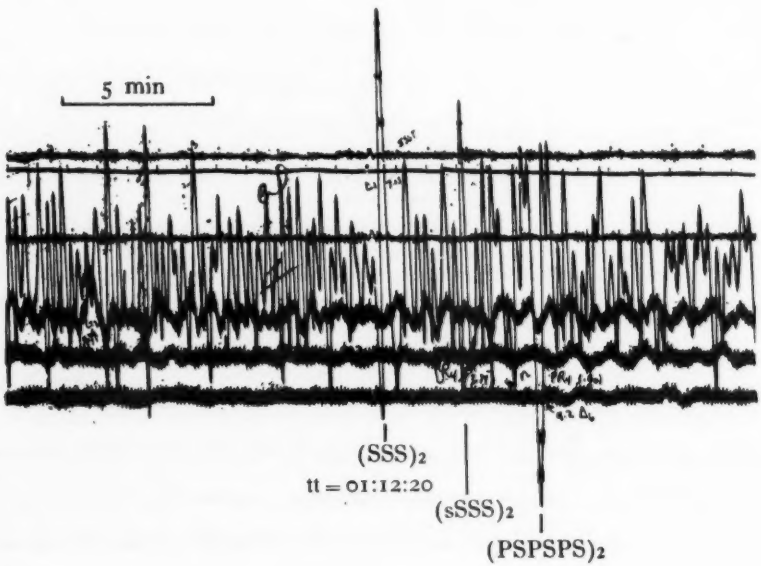

FIG. 10.—Multiple reflected body waves traversing major arc.
1956 May 23, 20 : 48 : 28 Fiji Islands $h = 400$ km
$M = 7\text{–}7\frac{1}{4}$ $\Delta_1 = 75\frac{1}{2}°$ $\Delta_2 = 284\frac{1}{2}°$
E–W, $T_0 = 30$ s, $T_g = 90$ s

4. Body waves

The *P* phase initiates a short period reverberation which makes it difficult to determine the character of later phases. Moreover crustal surface waves can mask the arrival of body waves which have traversed the major arc. Long period seismographs with reduced response to waves with periods less than about 20 s produce records of body waves which could not have otherwise been detected.

Unsuspected long period components in the spectrum of body waves are revealed. The pulse shapes of these phases exhibit simple characteristic forms which must ultimately be related to the mechanism at the focus.

FIG. 11.—Triple reflected *PS* traversing major arc.
Isabella Strain, T_g = 600 s, 1957 December 4.

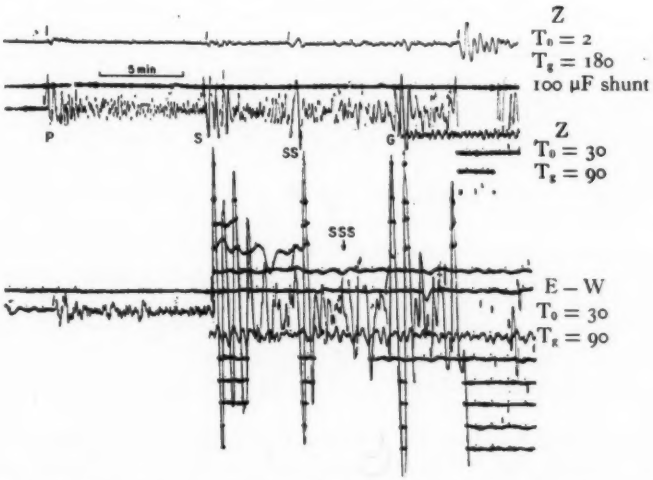

FIG. 12. Body waves recorded with two different long period systems.
1956 December 18, 02 : 31 : 00. Chile–Argentine border
$M = 7$–$7\frac{1}{4}$.

Figures 10 and 11 show multiple reflected body waves which have traversed the major arc. Figure 12 contrasts seismograms of body waves on different instruments. Note the simplicity in form and the long period components in the condenser shunted instrument with $T_0 = 2$ s, $T_g = 180$ s.

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The Magnetic Field and the Central Core of the Earth*

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Summary

The suggestion that the inner core of the Earth is liquid and is the seat of a strong magnetic field is examined. Contrary to expectations of a magneto-hydrodynamic rigidity for the inner core, it is shown that a liquid inner core with a strong magnetic field should not transmit shear waves. The absence of the phase *PKJKP* in seismic records is favourable to the hypothesis of a strong magnetic field in a liquid inner core. If the jump in *P* wave velocity at the inner core boundary is due to a magnetic field, the strength of the field must be of the order of 5×10^6 G whether the field geometry is toroidal or random. A strong toroidal field leads to an apparent ellipticity of the inner core. No such ellipticity is observed, so that if the field in the inner core is toroidal, it must be less than 5×10^5 G. A random field does not lead to an ellipticity effect and no limit can be set on the magnitude of a random field.

The energy required for a magneto-hydrodynamic rigidity in the inner core is excessive. The magnetic energy is equally divided between the inner and outer cores. The hypothesis of a strong magnetic field in the inner core cannot explain the observed short period variations of the surface field.

1. Introduction

A seismic discontinuity at a depth of 5 121 km separates the inner core of the Earth from the outer core. The necessity for such a discontinuity in interpreting the velocity distribution within the Earth was first pointed out by Miss Lehmann. Miss Lehmann (1936) showed that the observations of apparent *PKP* phases in the shadow zone could be explained by an inner core in which the *P* wave velocity was significantly higher than the *P* velocity in the outer core. The careful analyses of Gutenberg & Richter (1938) and Jeffreys (1939 a, b) firmly established the existence of an inner core having a *P* wave velocity higher than that of the outer core. The velocity in the inner core varies little and has an average value of about 11·2 km/s. The velocity in the outer core immediately adjacent to the inner core is in doubt. Jeffreys (1952) indicates that at a depth of 4 982 km the *P* velocity suddenly starts to diminish with depth and decreases until the inner core is reached where there is a jump in velocity from 9·4 to 11·16 km/s. Gutenberg (1951) shows a discontinuous change in the slope of the velocity curve at 5 121 km

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depth with a rapid increase of velocity to the value of 11·15 km/s. Though the details are uncertain there is definitely an increase of P velocity of about 10 per cent at a depth of about 5 121 km over the value at the 4 982 km depth.

The inner core contains only about 0·75 per cent of the Earth's volume. The total mass of the inner core is very uncertain since the inner core contributes only a small proportion of the total mass and moment of inertia of the Earth. Bullen's (1953) two models place the density at 12·3 and 22·3 g/cm^3, thus adequately emphasizing the uncertainties. It is very probable that the mass of the inner core is not more than two to three per cent of the mass of the total Earth.

Though the inner core represents but a small fraction of the Earth, the nature of the inner core plays an important role in the interpretation of the Earth's magnetic field and of the large-scale motions of the Earth. Birch (1940) suggested that the inner core represents a crystalline solid phase of the material of the liquid outer core where the latter is mostly liquid iron. Later workers (Bullen 1950 a, 1953 a, b; Jacobs 1953, 1954; Uffen & Misener 1954) make similar postulates. The principal difficulty in accepting this interpretation has been the fact that no shear wave, the phase *PKJKP*, has been observed to traverse the inner core. Bullen (1950 b, 1953 a, b) constructed travel-time curves for the purpose of detecting *PKJKP*. Bullen (1956), Hutchinson (1955) and Burke-Gaffney (1953) have all searched for this phase without success.

Before proceeding with the consideration of an alternative hypothesis we summarize the evidence leading to the suggestion that the inner core is solid.

1. The velocity, V_K, of a P wave in the outer core is fixed by the ratio of the incompressibility k to the density ρ, $V_K = (k/\rho)^{\frac{1}{2}}$. If the inner core were liquid this would require that the incompressibility undergo a sharp jump at the inner core–outer core boundary. Bullen (1949, 1950) suggests that the velocity distribution within the Earth is consistent with the hypothesis that k and $dk/d\rho$ vary smoothly even across the core–mantle boundary. A smooth variation of k across the outer core–inner core boundary is obtained if the inner core is solid with a P wave velocity $V_I = \{(k+\frac{4}{3}\mu)/\rho\}^{\frac{1}{2}}$, assuming a reasonable value for the rigidity μ. Birch (1952), using the observed P wave velocity and an extrapolated incompressibility for iron, finds a Poisson's ratio of 0·37 to 0·38 for the inner core. This value is perhaps high for most metals but the calculation neglects effects of composition, temperature and phase change.

2. The outer core is generally taken to be iron or iron–nickel. If the inner core were liquid, an element heavier than iron is required for a denser inner core. Generally accepted cosmic abundance tables show no abundant element heavier than iron or nickel.

3. The temperature gradient within the outer core is assumed to be adiabatic and therefore the change of temperature in the core is small. At the increased pressure existing at the inner core boundary the liquid phase of the outer core would no longer be stable relative to the solid phase assuming the melting point is raised by pressure.

Since the evidence on the inner core is principally qualitative, the nature of the inner core remains very much in doubt. The purpose of the present paper is to examine the inner core in the light of two recent suggestions. Cole (1957) presents the hypothesis that the inner core is liquid and is the seat of a magnetic field. The assumed field is sufficiently strong to impart a magnetohydrodynamic rigidity to the inner core. MacDonald & Knopoff (1958), on the basis of asymptotic relations for the Thomas–Fermi–Dirac equation of state and on abundance

arguments, show that silicon is probably a major constituent of the Earth's core. An alloy of approximate composition $(Fe,Ni)_2Si$ meets the requirements imposed by both the equation of state and abundance considerations.

2. Seismic evidence on a magnetic field in the inner core

We first investigate the limits set by seismic observations on the magnetic field residing in the inner core. The theory of the interaction of hydrodynamic and magnetic phenomena in an electrical conducting fluid has been developed by Alfvén (1950). Knopoff (1955) extended the theory to the interaction between elastic wave motions and a magnetic field in an electrical conductor. We review that part of the theory applicable to the interaction of a seismic wave with a magnetic field.

We consider a plane seismic wave incident upon a plane discontinuity between two liquids of characteristic elastic wave velocities α_0 and α_1. The angle of incidence of the plane wave is ϕ and the angle of emergence is θ; the wave approaches the discontinuity from the side of the fluid having velocity α_0. The fluid having a velocity α_1 is assumed to be electrically conducting with a finite conductivity σ and density ρ and is permeated by a uniform magnetic induction **B** parallel to the discontinuity. No magnetic field is assumed on side o.

The linearized equations of motion in the fluid case are

$$\left(\nabla^2-\mu\sigma\frac{\partial}{\partial t}\right)\mathbf{F} = \sigma\left\{\nabla\wedge\nabla\wedge\left(\frac{\partial\mathbf{U}}{\partial t}\wedge\mathbf{B}\right)\right\}\wedge\mathbf{B}$$

where

$$\mathbf{F}/\rho = \alpha^2\nabla\nabla\cdot\mathbf{U}-\partial^2\mathbf{U}/\partial t^2, \text{ (Knopoff 1955)}$$

U is the displacement vector of a particle and μ is the magnetic permeability. We assume a harmonic motion

$$\mathbf{U} = U_0\mathbf{e}\exp i\omega\{(x\sin\theta+z\cos\theta)/V-t\}$$

where the z direction is chosen normal to the interface and the x direction is taken in the direction of the magnetic field. **e** is a unit vector lying in the x–z plane. The complex phase velocity in the region containing the magnetic field is given by the solution to the equation

$$(V^2-\alpha_1^2)(V^2+iV_e^2) = V_h^2(V^2-\alpha_1^2\sin^2\theta).$$

V_e is the product of the frequency and the skin depth $V_e = (\omega/\mu\sigma)^{\frac{1}{2}}$. V_h is the velocity of the magnetohydrodynamic wave (Alfvén wave), $V_h = B/(\mu\rho)^{\frac{1}{2}}$. Solving for the phase velocities under the constraint of Snell's law and assuming a high electrical conductivity $V_e \ll \alpha_1$, we obtain two complex phase velocities

$$V_1 = \{\alpha_1^2+V_h^2[1-(\alpha_1/\alpha_0)^2\sin^2\phi]\}^{\frac{1}{2}}$$

and

$$V_2 = \left\{\frac{i\alpha_1^2V_e^2}{\alpha_1^2+V_h^2[1-(\alpha_1/\alpha_0)^2\sin^2\phi]}\right\}^{\frac{1}{2}}.$$

The significant feature of this solution is that the second mode attenuates within its wavelength and would not be observed at any distance. Cole suggested that a strong magnetic field would lead to a shear wave because of the magnetic rigidity

and that, even though the inner core were fluid, the phase J should be observed assuming the inner core were the seat of a strong magnetic field. This conclusion is incorrect. The phase J should never be observed even in the presence of a magnetic field. This feature of the solution is readily understandable on physical grounds. The magneto-hydrodynamic waves travel with velocity V_h, are propagated along the lines of force of the magnetic field, and are polarized transverse to the lines of force. The field is parallel to the plane discontinuity. By Snell's Law, the ray is bent toward the normal since V_h is ordinarily less than α_0. These two conditions on the ray are inconsistent and the severe constraint imposed on the signal leads to rapid attenuation.

The prediction that the phase *PKJKP* should never be observed if there is a magnetic field in a liquid inner core is consistent with observations. This feature is an advantage of an earth model with a liquid inner core over a model with a solid inner core.

The jump in P wave velocity at the 5 121 km discontinuity may give us some information concerning the magnitude of the magnetic field. We first assume that the inner core has the same composition as the outer core. At the interface, the elastic velocities are equal

$$\alpha_1 = \alpha_2 = \alpha$$

The velocity in the inner core is

$$V_I = (\alpha^2 + V_h^2)^{\frac{1}{2}}$$

where we neglect the problem of the variation of the magneto-hydrodynamic term with polarization and angle. At the bottom of the outer core the velocity is $V_K = \alpha$. The observed jump in velocity can be attributed to the presence of a magnetic field perturbing the P wave velocity in the inner core. If we assume the velocity-depth curves are computed for normal incidence, then using $V_I = 11{\cdot}2$ km/s, and $V_K = 10{\cdot}5$ km/s, the induction B in the inner core is 5×10^6 G where the jump in velocity is due entirely to the presence of a magnetic field. The magneto-hydrodynamic velocity $V_h = 4$ km/s is a value consistent with that assumed by Cole, though Cole underestimated the needed magnetic induction by an order of magnitude. If the minimum velocity noted by Jeffreys in the region between 4 982 km and 5 121 km depth is associated with a field free region, the jump in velocity to the value at 5 121 km can only correspond to a field far in excess of 5×10^6 G.

The calculation of a magnetic induction depends on the assumption of identical composition across the discontinuity. Under this condition there is no obvious mechanism whereby a large magnetic field could be concentrated in the inner core and not be found in the outer core. One means by which a magnetic field could be stored in the inner core is to make the outer core a poorer electrical conductor than the inner core. The magnetic field will reside in the better conductor because of the smaller ohmic losses therein. A difference in chemical composition between the outer and inner core could produce the needed discontinuity in electrical conductivity. MacDonald & Knopoff (1958) show that a bulk composition of the core of $(Fe,Ni)_2Si$ is consistent with a chondritic meteorite model of the Earth and with the representative atomic number as fixed by seismic observations and the asymptotic relations for the Thomas–Fermi–Dirac equation of state. The chondritic meteorite model is in turn consistent with the heat flow measurements. In the presence of a gravitational field, a liquid of bulk composition $(Fe,Ni)_2Si$ might be

expected to separate with iron and nickel concentrated inwardly relative to silicon; in turn nickel concentrates inwardly relative to iron. Such a separation could provide a conductivity difference of one order of magnitude between the inner and outer core with the silicon rich outer core having a lower conductivity than the nickel rich inner core.

If the compositions of the inner and outer core differ, we can no longer take $\alpha_0 = \alpha_1$. The variation of P wave velocity with atomic number is indicated by the asymptotic relations for the Thomas–Fermi–Dirac model (March 1955; Gilvarry 1957). We should expect that α_0 should be $2\frac{1}{2}$ to $3\frac{1}{2}$ per cent higher than α_1 if there is a difference of 3 to 4 atomic numbers for the materials in the outer and inner core. Since $\alpha_1 < \alpha_0$, under the assumption of higher silicon content in the outer core, the estimate of the magnetic induction of 5×10^6 G required to produce the P velocity jump is too low. Alternatively if a smaller field of less than 5×10^5 G is postulated for the inner core, then at pressures of 10^6 atm the velocity in liquid iron–nickel is greater than the velocity in iron silicide in opposition to the predictions of the asymptotic relations. We note that at one atmosphere the velocities in elements Fe, Cu, Zn and copper–zinc brasses have the order predicted by the asymptotic relations but Ni does not.

3. Apparent ellipticity of the inner core due to a magnetic field

A further test of the existence of a magnetic field in the inner core results if we suppose that the field has a toroidal geometry. Bullard & Gellman (1954), and Elsasser (1950, 1955) postulate that the field is mainly toroidal. A second model by Cole pictures the field lines as turbulent or randomly non-radial. The poloidal field is here ignored as being of only a secondary nature. The magnitude of the poloidal field at the surface of the inner core is about 60 G. Elsasser suggests a conversion efficiency between poloidal and toroidal fields of the order of 10^3 so that a toroidal field of the order of 6×10^4 gauss in the inner core is a possibility. Alternatively if the jump in seismic velocity at the inner core boundary is due to a magnetic field, then the strength of the field in the inner core is at least 5×10^6 G regardless of the toroidal or turbulent character of the field. It should be noted that if the field is turbulent, large Maxwell stresses in excess of 10^4 atm must be maintained against only the viscosity and Coriolis forces.

If the field is toroidal then the angular dependence of the phase velocity of *PKJKP* is important. In the case of a diametral ray through the equator of the toroid, the velocity is $(\alpha_1^2 + V_h^2)^{\frac{1}{2}}$. A ray on the axis of the toroid has a velocity α_1. The variation in velocity is equivalent to an ellipticity of a homogeneous inner core. The difference in velocity between an equatorial and axial ray is $V_h^2/2\alpha_1$ for $V_h \ll \alpha_1$. A uniform magnetic field of 5×10^5 G throughout the inner core would lead to a difference in travel times of axial and equatorial rays of 0·6 s, a time difference too small to be observed. A field of 5×10^6 G produces a maximum ellipticity correction of 50 s. A rough investigation of the times of *PKIKP* has shown no such ellipticity. We infer that seismic travel times place an upper limit to the magnitude of the magnetic field, provided the field is toroidal. No such limit is possible if the field is random.

A further bit of evidence on the inner core is Gutenberg's (1957) observation of a precursor to *PKIKP* with a time lead of 12 to 15 s. This precursor might be interpreted as a consequence of oblique incidence upon the inner–outer core boundary. We resolve the incident ray onto the plane containing the magnetic

field and the normal to the discontinuity and onto the plane normal to the magnetic field. The first component traverses the inner core with velocity

$$\{\alpha_1^2 + V_h^2[1 - (\alpha_1/\alpha_0)^2 \sin^2\phi]\}^{\frac{1}{2}}$$

while the other component travels with a greater velocity $(\alpha_1^2 + V_h^2)^{\frac{1}{2}}$. The travel-time difference between the two components is 12 s at $\Delta = 140°$ provided the magnetic induction is of the order of 5×10^6 G. However, Gutenberg points out that the precursor is a high-frequency component with a period of about 1 s while the main *PKIKP* phase has a period of about 3 s. Since the velocity V_1 is not a function of frequency, the precursor must originate elsewhere than in the magnetic field, unless diffraction in the presence of a complicated geometry provides the observed dispersion.

4. Geophysical consequences

The hypothesis of a strong magnetic field in a liquid inner core has several geophysical consequences. First, since the inner core is now presumed liquid instead of solid, the temperature in the inner core is now above the melting point for iron at the pressure of the core. None of the difficulties obtained by Jacobs (1954) and Uffen & Misener in reconciling the assumed solidity for the inner core with the melting point information (Simon 1953) and the temperature gradients in the outer core (Valle 1952) obtains in the present case. The central temperature can now be specified to be in excess of 4 000° K.

The magnetic time constant $4\pi\mu\sigma a^2$ determines the lifetime of the magnetic field; a is the radius of the body. Reducing the radius of the magnetic region from that of the outer core to that of the inner core reduces the time constant given by Elsasser (1950) to about 5 000 years. If we state that circulatory motions are not expressly forbidden in the outer core, the hypothesized reduction in conductivity of the outer core reduces this magnetic time constant to about the same value. Thus if we started with an initial budget of mechanical energy in both cores the magnetic energies resulting from the circulation would be equally partitioned in the inner and outer cores. Thus some other mechanism, probably of a mechanical type, must act in addition to the electrical mechanism in order to store a strong magnetic field in the inner core.

The surface magnetic field undergoes short period variations with a period of 500 years (Chapman 1951). Let us assume that these variations originate in the inner core. If the time constant for the decay of a field in the outer core is 5 000 years, then the fluctuating poloidal field at the surface of the inner core must be e^{10} greater than that needed to produce the observed fluctuations in the absence of an outer core. In order for these short period variations to originate in the inner core, the non-poloidal field in the inner core must vary in magnitude by at least 10^7 G. There is no seismic evidence for fields in excess of 10^7 G. The jump in P wave velocity at the inner core boundary requires a magnetic field in the inner core of the order of 10^7 G. If the short period variations are to originate in the inner core a still stronger total field is required. The fact that the inner core shows no seismic ellipticity rules out a toroidal field of this strength. The energy required to maintain a random field of this magnitude is very large and there are no apparent sources for this energy. It thus seems unlikely that the inner core is liquid.

5. Summary

A model in which a liquid inner core is permeated by a strong magnetic field is not inconsistent with seismic observations or with abundance arguments. In such a model, the phase *PKJKP* has a vanishingly small amplitude and should not be observed. The phase *PKJKP* has not been found. Because of the small energy associated with this phase (Bullen 1951), it should be most difficult to detect, so that the absence of *PKJKP* cannot as yet be regarded as positive evidence for a liquid core. The discontinuity of the *P* wave velocity at the inner core, if interpreted solely as due to a magnetic field, requires a field of 5×10^6 G in the inner core whether the field geometry is toroidal or random. Observations of the lack of angular dependence of the velocity of the phase *PKIKP* rules out a field greater than 5×10^5 G if the field is toroidal. No limit can be set if the field is random. The *PKIKP* precursor of Gutenberg is not inconsistent with a field of 5×10^6 G in the toroidal model but should not be observed in the random model. The random model for the field requires Maxwell stresses high compared with probable viscous stresses and Coriolis forces. If the field in the inner core is toroidal, it must be less than 5×10^5 G. No limit can be set if the field is random but a figure of 5×10^6 G is indicated. These fields are too small to explain the observed fluctuations of the magnetic field; the inner core is very likely not liquid.

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Palaeogeographic Reconstruction from Palaeomagnetism

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Summary

A method for constructing maps of the positions of the continents in the past, using palaeomagnetic results, is described. It is illustrated by using results from Europe, Australia, India, Africa and South America to provide possible reconstructions of the past positions of these continents.

1. Introduction

There are now good reasons for supposing that the directions of magnetization in some rock formations reflect the directions of the Earth's magnetic field at the time of deposition (Graham 1949, Runcorn 1955, Irving 1957 b). The Earth's magnetic axis relative to the present coordinates may be calculated from palaeomagnetic data on the supposition that the geomagnetic field when averaged over several thousands of years approximates to that of a geocentric dipole. This has been the case since the middle of the Tertiary (Hospers 1955, Irving & Green 1957) and the reasons for supposing an average dipole field for periods prior to this are strong (Runcorn 1954, Creer, Irving & Runcorn 1957, Irving 1957 a). With a few exceptions the pole determinations made in this way never coincide with the present pole if the rocks from which they were obtained are older than Miocene, and this is always so for Triassic or older rocks. Also determinations from Pre-Miocene rocks of comparable age from different continents do not agree. These divergences from the present pole and from one another can be interpreted in terms of polar wandering and continental drift.

For the present purposes polar wandering is defined as incremental movements of the dipole axis relative to the crust as a whole, and continental drift as horizontal movements of these continents relative to one another. The former is a general phenomenon and would be reflected in the palaeomagnetic results everywhere. The second involves special movements of each continent and would cause differences in the palaeomagnetic results from different continents.

Mechanically, polar wandering is thought of as movement of the whole Earth relative to the axis of rotation which, aside from precession and nutation, remains fixed relative to the Sun. It seems that this could arise from redistribution of mass due to geological processes in the crust or convection in the mantle (Vening Meinesz 1948, Gold 1955, Munk 1956). It may be assumed that the rotational and dipole axes have always been coupled together, as they appear to have been in the past 20 million years, so that experimentally determined movements of the

dipole axis may be identified with movements of the Earth as a whole relative to the geographic axis. However, this is unnecessary in the present context. Continental drift has been regarded as mechanically implausible (Jeffreys 1952) although this may not necessarily be the case (Scheidegger 1957). Apart from the palaeomagnetic evidence the case for its occurrence rests on morphological, geological and biological grounds. Proponents of continental drift suggest that in Carboniferous, Permian and Triassic times there was a single land-mass, Pangea, or twin continents, Laurasia and Gondwanaland. These later broke up into the seven continents which subsequently drifted to their present positions. So far as palaeomagnetism is concerned the validity or otherwise of this process depends primarily on results from Carboniferous and younger rocks and therefore only these are discussed.

Attempts have already been made to compare the palaeomagnetic results with previous theories of polar wandering and continental drift (Creer & others 1957, Irving 1957). It is more satisfactory to prepare maps of the continental dispositions and shifts of the polar axis based *entirely* on the palaeomagnetic results. Some preliminary attempts at this have been made (Du Bois 1957, Irving 1957 a). This paper gives a graphical method for producing such reconstructions, the principle being to assume one continent and its poles fixed and to move other continents in order to unify equivalent poles. As illustrations, results from Europe, Australia, Africa, South America and India are used to prepare maps showing possible relative positions of these in the past. These reconstructions are meant as an explanation of the palaeomagnetic data only, and are offered without prejudice as to their mechanical or geological probability. The reality or otherwise of continental drift depends on observations, not on theory, so that it is unwise to discount it on the grounds that an adequate mechanism is not known. Similarly, the interpretation of the palaeomagnetic data in terms of continental drift and polar wandering is not dependent on the geological and biological data whose interpretation by different authors is equivocal.

Stereographic nets are used in all the figures, polar projections in Figures 1 and 3 to 7 and equatorial in 8 and 9.

2. Polar movement relative to the continents

Results are now available from all continents except Antarctica (Figure 1). For North America, Europe and Australia the path back to the Proterozoic is known. It has been obtained for India back to the Jurassic. Each path is related only to the continent from which it was obtained and represents the movements of this continent relative to the geomagnetic axis. Both polar wandering and continental drift could contribute to these movements. For Africa and South America several pole positions have been calculated but are inadequate to draw paths.

The paths for North America, Europe and Australia have the same general form concave to the east. They pass through similar latitudes in the Carboniferous, Permian and Triassic and travelling with similar velocities reach the present pole at about the same time. However, the paths differ in position by 170° of longitude equivalent poles being displaced longitudinally from one another. The Carboniferous and Permian poles are approximately 90° of arc from Europe and North America whereas equivalent poles from Australia are near to Australia, despite the fact that nowadays the latter is antipodal to the two northern continents.

As an initial approximation these polar movements were represented as smooth

curves (Creer & others 1957, Irving & Green 1958). However, many irregularities are now becoming apparent. For instance, the Carboniferous poles are often in advance of the Permian poles (that is, nearer the present pole). The smoothed curves could be the averaged effect of a large number of polar movements which

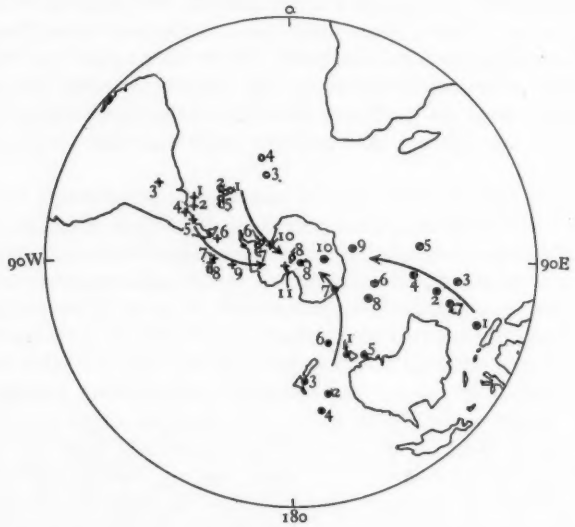

FIG. 1.—Pole positions calculated from Australia, Europe, India and North America. The poles are indicated by dots, circles, circles with central dots, and crosses respectively. They are numbered as follows for Australia (Irving & Green 1958): 1 Kuttung lavas, Carboniferous; 2, Kuttung sediments; 3, Permian Lower Marine basalt; 4, Permian Upper Marine basalt; 5, Brisbane tuffs, Triassic; 6, Tasmanian dolerite sills; 7, Older volcanics of Victoria; 8, Newer Volcanics of Victoria.

The European poles have these numbers; 1, Carboniferous lavas and sediments (Belshé 1957): 2, Estérel lavas (obtained by averaging results from flow R4 and a dolerite) (Roche 1957) and flow R3 (Rutten, Everdingen & Zijderveld 1957); 3, Lavas of the Oslo graben (Rutten & others 1957); 4, Exeter traps (Creer 1957); 5, Keuper marls and sandstones (Clegg, Almond & Stubbs 1954); 6, Antrim basalts (Hospers & Charlesworth 1954); 7, Oligocene dykes in France (Roche 1950); 8, Miocene lavas Iceland (Hospers 1955).

The poles from India have these numbers: 1, Rahajmahal traps (Clegg & others 1958); 2, Deccan traps (Irving 1956); 3 to 10, Lavas from various places in the Deccan traps (Clegg & others 1956, 1957); 3, Khandala, 4, Linga, 5 and 6, Nipani lower and upper, 7 and 8 Amba lower and upper, 9 and 10 Pavagadh lower and upper.

The North American poles are numbered as follows: 1, Barnett shale, Mississippian (Howell & Martinez 1957); 2, Naco sandstone, Pennsylvanian (Runcorn 1956); 3, 4 and 5 Supai shales (Runcorn 1956, Doell 1955 and Graham 1955 respectively); 6, 7, 8 and 9 Triassic formations (Du Bois, Irving, Opdyke, Runcorn & Banks 1957); 10, Dakota sandstone (Runcorn, 1956); 11, Columbia River basalts (Campbell & Runcorn 1956).

were irregular in magnitude, timing and direction. Runcorn (1957) and Green (1958) have suggested that polar movements may have the form of a random walk on a sphere. It is not yet clear to what extent the observed irregularities are due to errors in determinations or stratigraphic correlation, or to exceptionally rapid polar wandering within a geological period, or to temporary departures from a dipole field.

3. Continental reconstructions

Considering continental drift in this context it is assumed that each continental block behaves as a separate entity. Translations or rotations may occur. Rotations may be regarded as occurring round the centroid of area of a continent. Translations are conveniently considered as either latitudinal (change in latitude without change in longitude) or longitudinal (change in longitude without change in latitude). Appreciable latitudinal components, as well as rotations, would contribute additional positive or negative velocities to the polar curves, different in each continent, and destroy their comparable features. In North America, Australia and Europe the similarities in form and velocity suggest a common geophysical effect, namely polar wandering, and that latitudinal components and relative rotations have been small. The differences in position of these polar curves suggest large longitudinal movements between the land-masses concerned. In the case of India latitudinal movements also appear to have been important. Since the Jurassic the pole relative to India (Figure 1) appears to have travelled a latitudinal distance of 77°. In contrast in South America where the results also extend back to the Jurassic the divergences from the present pole are small (Creer 1958) and it is possible that a southward latitudinal drift component or an anticlockwise rotation has suppressed it.

The problem is to express these land movements cartographically. This may be done by calculating the continental movements necessary to unify equivalent pole determinations, equivalent poles being defined as those calculated from rock formations which are contemporaneous in a geological sense. It is assumed in this procedure that equivalent poles are comparable; or, more precisely, that the rock formations from which the groups of poles were obtained were laid down during, but not necessarily covering the whole of a time interval during which polar wandering was negligible or smaller than the experimental errors. This implies comparisons between sub-divisions of geological periods, that is, between series and epochs. This ought to be possible eventually, but at the present stage only comparisons between poles obtained as an average for a period can be made. Even with finer sub-division of geological time, errors will always arise from this source, but these will be lost when comparisons between many groups of equivalent poles are made, since errors in age correlations between continents are not likely to be biased one way. In other words it is equally probable that $(t_a - t_b)$ is positive or negative where t_a and t_b are the absolute ages of two rock formations from different continents and which are regarded as geologically contemporaneous.

Because of the occurrence of reversals of magnetization it is often difficult to identify the calculated pole as north or south, and equivalent poles from different continents must always refer to the same pole. The post-Carboniferous polar curves are continuous with the present geographic poles and this ambiguity does not arise, although it will when results from the lower Palaeozoic and Pre-Cambrian are considered.

First assume continent A with successive pole positions a_1, a_2, a_3 . . ., for the time interval 1, 2, 3, . . ., fixed relative to the crust of the Earth. Now allow continent B, for which there are pole determinations b_1, b_2, b_3, . . ., for the same time intervals to move so that b_1 coincides with a_1, b_2 with a_2 and so on. The co-latitudes p_1, p_2, p_3, . . ., for B during geological periods 1, 2, 3, . . ., can be calculated from the magnetic inclination I_1, I_2, I_3, . . ., measured from rock formations laid down during these periods by the equation

$$\cot p = \tfrac{1}{2} \tan I.$$

The measured declination does not define the longitude so that the possible positions of B are represented by a family of small circles of radius p_1, p_2, p_3, . . ., and centres a_1, a_2, a_3, ...

In theory, the choice of a_1, a_2, a_3, . . ., is arbitrary; for instance, they may be chosen to coincide, but in practice it is preferable to take experimentally determined poles from continent A. Since the angular distances between successive pole positions a_1, a_2, a_3, . . ., is usually very much smaller than the co-latitudes p_1, p_2, p_3, . . ., these small circles will intersect. Continents C, D, E, . . ., with experimentally determined co-latitudes for the same time intervals can now be treated in the same way as B. If continent A has not moved relative to the Earth's crust then the polar path a_1, a_2, a_3, . . ., will be due to polar wandering so that the reconstructions will show the relative movements of the continents only, otherwise motions common to all the continents will be included. Correction for this will be a simple matter, but will not be possible until very many more data are available.

Figure 2 represents two possibilities that could arise. Figure 2a illustrates the

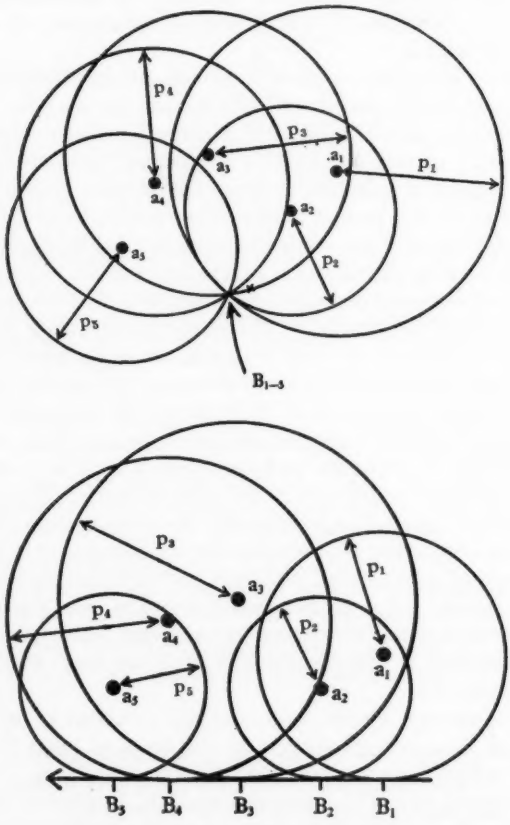

FIG. 2.—Continental Reconstructions.
Poles a_1 to a_5 for the time intervals 1 to 5 and continent A from which they were calculated are assumed fixed. Around these, circles of radius p_1 to p_5 are constructed, p_1 to p_5 being the experimentally determined co-latitudes for continent B for the same time intervals.

case of polar wandering with two continents A and B which do not move relative to one another. Continent A and its poles are assumed fixed and the common intersection point of the co-latitude circles represents the most likely position for continent B during the time intervals considered. The intersection point may not be the present position of B suggesting that B has moved relative to A subsequently. In Figure 2b the case of concurrent polar and relative continental movement is illustrated. Again assuming A fixed, the simplest path followed by B is the envelope containing the co-latitude circles. The full interpretation of these diagrams depends fundamentally on the manner in which polar wandering occurs—whether it is uniform or irregular—and if it is irregular on the degree of irregularity. This is not known at present. Figure 2 is simply meant to show that although for any one geological period an infinite number of continental arrangements will satisfy the palaeomagnetic data, the occurrence of polar wandering between successive geological periods could restrict this number considerably. Using the additional restriction that the super-imposition of continents is absurd and working back from present-day geography, it should be possible when polar sequences are available from all continents to draw reconstructions of past continental positions.

4. Movement of Australia

As an illustration of this method palaeomagnetic results from Australia and Europe are used. The poles (south) calculated from Europe for the Carboniferous, Permian, Triassic and Eocene are plotted in Figure 3. The Jurassic pole is obtained by interpolation. The equivalent poles from Australia are also given in Figure 3. The poles (see Table 1) are labelled a_1 to a_5 and b_1 to b_5 respectively, the labelling being consistent in Figures 3 to 9. Assuming the European poles fixed, the position of Australia is defined by small circles around these poles of a

Table 1

Pole positions from Europe and Australia

	Europe			Australia		
	No.	Lat.	Long.	No.	Lat.	Long.
Eocene	a_5	74° S	47° W	b_5	67° S	123° E
Jurassic	a_4	(60° S)	(49° W)	b_4	50° S	157° E
Triassic	a_3	43° S	49° W	b_3	39° S	143° E
Permian	a_2	46° S	24° W	b_2	32° S	171° E
Carboniferous	a_1	47° S	43° W	b_1	37° S	158° E

The poles are numbered as follows, the southern hemisphere positions only being given:

a_1 Carboniferous of England (calculated from the results of Belshé 1957)
a_2 Permian of Europe, obtained as a mean of results from England (Creer 1957), Norway (Rutten & others 1957), and France (Roche 1957, Rutten & others 1957)
a_3 Keuper Marls and Sandstones (Clegg & others, 1954)
a_4 pole by interpolation
a_5 Antrim basalts (Hospers & Charlesworth 1954)
b_1 mean pole from sediments and lavas of the Kuttung Series
b_2 mean pole from lavas of Upper and Lower Marine series in New South Wales
b_3 Brisbane tuffs
b_4 Dolerite sills of Tasmania
b_5 Older volcanics of Victoria.

radius equal to the co-latitude calculated from the palaeomagnetic data. In figure 4 these small circles are drawn for a particular place in Australia, namely Alice Springs (Figure 3). Figure 5 gives possible positions for Australia as a whole for all five periods. Figure 5 has been obtained graphically by rotating outlines of Australia around the equivalent European poles. This has been done on a globe 19 in. in diameter. Similar methods have been used to construct the maps in Figures 8 and 9. The accuracy achieved is about 3° which is better than the accuracy of pole determinations and therefore adequate for the present purposes. The intersection of the circles in the region of 60° W 30° S in Figure 4 and the occurrence of common positions for Australia shown by the full outlines in figure

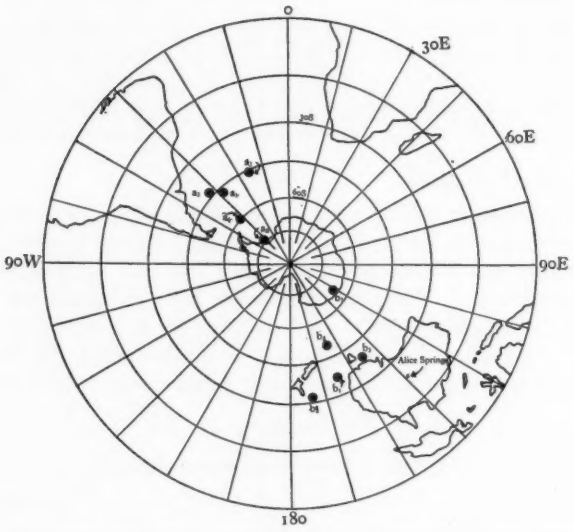

FIG. 3.—Mean pole positions for the Carboniferous, Permian, Triassic, Jurassic and Eocene from Europe and Australia (Southern Hemisphere). The poles are numbered as in Table 1.

5 are noteworthy. The simplest interpretation is that in Carboniferous to Eocene times Australia was situated in what is nowadays the position of southern South America (Figure 6 positions 1 and 2) and has subsequently moved to its present position (Figure 6 position 4). Another interpretation is that the movement commenced earlier, between the Jurassic and Eocene so that position 3 (Figure 6) is intermediate. Other much more complicated movements of Australia which involve considerable rotations and frequent changes in the direction of translation and sense of rotation are possible and may be visualized from the dotted outlines in Figure 5. Such alternatives regard the coincidences shown in Figures 4 and 5 as fortuitous.

Of course it is an equally valid procedure to assume Australia and the Australian poles fixed and allow Europe to move. The co-latitude circles for London around b_1 to b_5 are given in Figure 7, and the intersection is now in Eastern Siberia. In Figures 4 and 7 the relative positions of London and Alice Springs is the same, their distance apart being approximately 90° of arc.

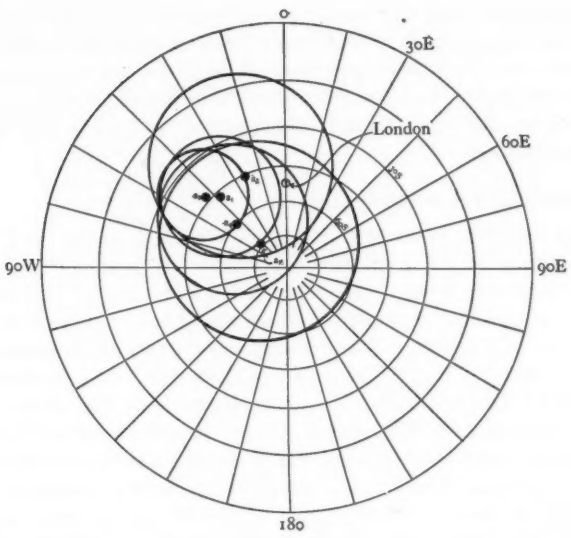

FIG. 4.—The co-latitude circles for Alice Springs (Southern Hemisphere). The European poles are labelled as in Table 1 and are assumed fixed. The radii of these circles are the palaeomagnetic co-latitudes for Alice Springs. The position of London is marked in the northern hemisphere.

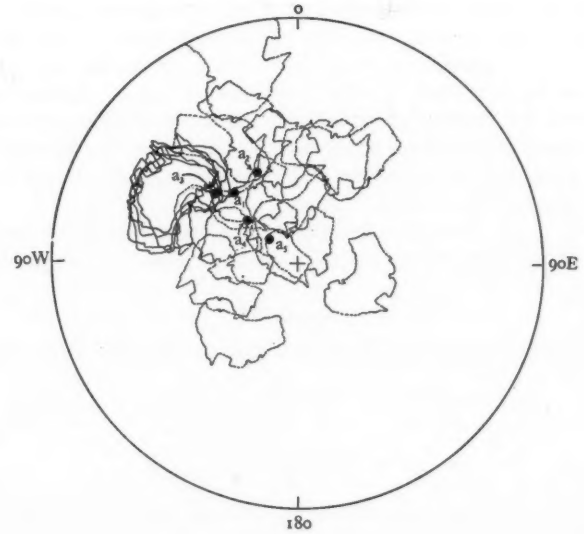

FIG. 5.—Possible positions for Australia relative to the poles calculated from Europe (Southern Hemisphere). The poles are labelled as in Table 1.

These reconstructions are open to many criticisms. For instance, the pairs of poles here compared may not satisfy the definition of equivalence given above. In addition, the Jurassic pole from Europe has been obtained by interpolation, not by experiment, and the Triassic pole from Australia is not yet accurately defined, (Irving & Green 1958). Such criticisms cannot be fully met at present so that the interpretations in Figure 6 require confirmation or rejection by future palaeomagnetic observations.

5. Gondwanaland reconstructions

At present the relevant results from Gondwanaland are few so that many possible arrangements of the southern continents will satisfy the palaeomagnetic results. Two solutions (I and II) are given in Figures 8 and 9. The European poles are fixed and the position assumed for Australia during Carboniferous to Eocene times is position 1 in Figure 6. The results from the southern continents are these. For Africa the pole (89° E, 68° S) for Jurassic time has been obtained by averaging results from the Karroo dolerites. These results are from the Victoria Falls (Nairn 1956), from the mines of Winkelhaak in the Transvaal (upper and lower sills) and of Escourt in Natal (both normal and reversed), and from surface rocks in the Cape Province, Orange Free State, Transvaal and Natal (Graham & Hales 1957). For the Jurassic of South America the pole (54° W, 83° S) from the Serra Geral formation is used (Creer 1958). These results give the co-latitudes for Africa and South America in the Jurassic and the usual circles can be constructed with the equivalent European pole (a_4) as centre. Likewise for India the co-latitude in Eocene times is given by the pole result from the Deccan Traps (98° E, 28° S, obtained as an average of results by Clegg & others (1956) and Irving (1956)) and the co-latitude circle around the European Eocene pole (a_5).

Figures 8 and 9 are presented to illustrate the usefulness of these graphical methods for the study and prediction of palaeomagnetic results. From such reconstructions the average direction of magnetization, both inclination and declination, for any place at a given time (and therefore for any particular rock formation) can be predicted. Forthcoming results can be checked against these predictions and the number of possible continental arrangements thereby systematically reduced. A thorough study of such predictions could yield information as to which are the more important rock formations to study, and could be very useful when future work is being planned.

The value of the average magnetic inclination predicted by reconstructions I and II for several periods are given in Table 2. The recently published results

Table 2

The magnetic inclination predicted by Gondwanaland Reconstructions I, II

	Carboniferous		Permian		Triassic		Jurassic		Eocene	
	I	II	I	II	I	II	I	II	I	II
South America	66	54	72	39	67	56	—	—	45	60
India	46	72	56	64	36	75	50	67	—	—
Africa	62	57	71	44	56	63	—	—	53	49

The values are calculated for the following regions: South America—Rio Grande do Sul, India—Bihar, Africa—Durban. The blank columns are the experimental values used in the reconstructions.

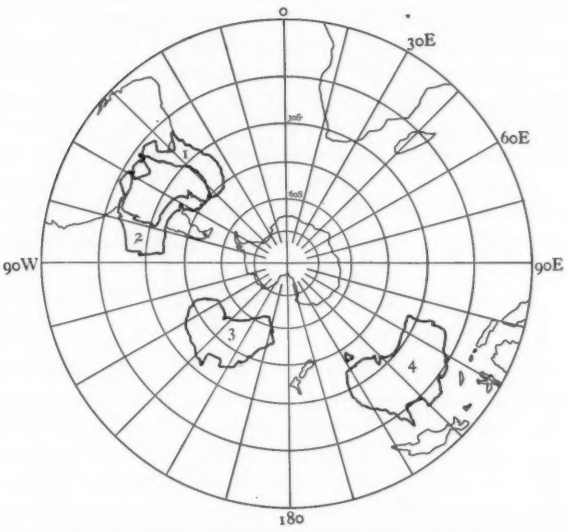

FIG. 6.—The most likely positions for Australia given by the palaeomagnetic data (Southern Hemisphere).
Positions 1 and 2 are taken from the coincidence zone in Figure 5 for the Carboniferous, Permian, Triassic, Jurassic and Eocene co-latitude circles. Position 3 is from the Eocene co-latitude circle and could be an intermediate between 1 and 2 and the present position 4. The outlines of the present continents are added in thinner line.

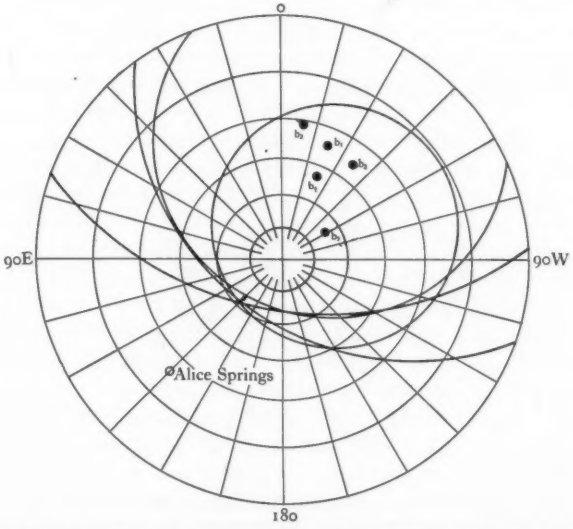

FIG. 7.—The co-latitude circles for London (Northern Hemisphere). The Australian poles are labelled as in Table 1, and assumed fixed. The radii of the small circles are the palaeomagnetic co-latitudes for London. The position of Alice Springs is marked on the southern hemisphere.

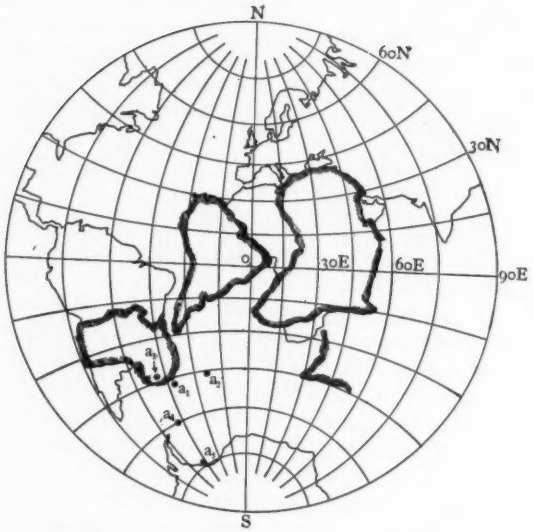

FIG. 8.—Reconstruction of Gondwanaland I.
This map of the Eastern Hemisphere shows one of the numerous possible arrangements of the southern continents (in heavy line) consistent with the palaeomagnetic results. The present continental positions are added in thinner line and the poles are numbered as in Table 1.

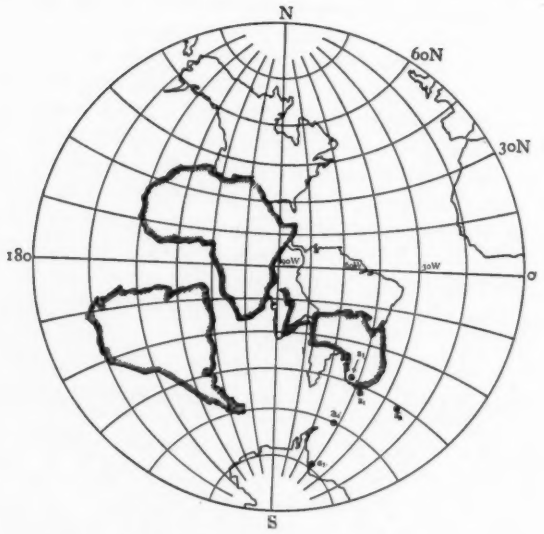

FIG. 9.—Reconstruction of Gondwanaland II.
A map of the Western Hemisphere showing another possible arrangement of the continents (in heavy line). The present continental positions are added in thinner line and the poles are numbered as in Table 1. Two positions for India are shown, the one heavily outlined obtained from the Deccan traps, the other from the Rahajmahal traps.

from the Jurassic Rahajmahal traps of India are relevant in this connection (Clegg & others 1958). The measured value of dip is 64° fitting with the predicted value of 67° for reconstruction II, but not the other. A position for India consistent with the measured direction of magnetization is given in Figure 9. From the Northern Hemisphere data the Carboniferous appears to be a period of rapid polar wandering (Everitt, reported in Clegg & others 1957), and for this reason the Permian, Triassic and Jurassic are more suitable periods from which to select rock formations to test these reconstructions.

It is of some interest that the reconstruction in Figure 9 is substantially similar to the Gondawanaland maps published by Wegener in 1924 (see for instance Figure 23).

6. Discussion

The methods given above provide a means of constructing maps showing the relative positions of the continents in the past from the palaeomagnetic results. The author wishes to stress that the reconstructions given in Figures 6, 8 and 9 are intended only as *illustrations* of these methods. Such reconstructions represent some of the ways of explaining the palaeomagnetic results, but this is not to say that other explanations are not possible. Three important assumptions are made; first, that the directions of magnetization in some rock formations reflect the direction of the Earth's magnetic field at the time of deposition, secondly, that the geomagnetic field when averaged over several thousands of years approximates to that of a geocentric dipole, and lastly, there is the assumption regarding equivalence of poles. The evidence for the first two suppositions has been referred to in the introduction. The assumption of polar equivalence depends primarily on the accuracy of inter-continental geological correlations and the density of palaeomagnetic data. It could be argued that because of correlation errors and the paucity of data the assumption at the present stage of polar equivalence is invalid. The increased use of isotope decay schemes for dating rocks should eventually eliminate these correlation errors, and the rapid accumulation of palaeomagnetic data ought soon to remedy the sampling inadequacies.

Because a single palaeomagnetic result does not fix the ancient longitude of a continent it was thought at first (Irving 1957 a) that many widely differing reconstructions would be possible. However, if continental drift has happened as periods of movement separated by defined standstills during which polar wandering occurred (Figure 2 a), then other coincidences like those for Australia (Figure 5) will arise for all continents. If such coincidences should happen frequently the probability of their being significant becomes very great. It is not unlikely that relative longitudes will be more closely defined in this way, and a very limited number of solutions, perhaps even a unique solution, may arise. The method could be stated in mathematical terms, but with the present experimental accuracies the graphical methods used here are adequate.

Reconstructions comparable to those in Figure 8 and 9 can be obtained by assuming Australia and the Australian poles fixed and moving the other continents accordingly. The assumption of different fixed poles does not alter the relative positions of the continents obtained, but it does affect the positions of the continents relative to their present positions. All these reconstructions (many more have been drawn) have one feature in common—that with the exception of India the longitudinal movements implied by the palaeomagnetic results far exceed those

in a latitudinal direction. (This has already been deduced from general considerations in Section 3). For instance, the movements required to translate the continents from the positions given in Figure 9 to their present positions can be calculated from this figure. Expressed in terms of longitudinal and latitudinal components of movement they are for South America 70° and 5° or 7 100 and 600 km respectively. The equivalent figures for Australia are (188°, 0°) (18 100 km, 0 km), for Africa (135°, 0°) (14 900 km, 0 km), and for India (162°, 41°) (17 200 km, 4 500 km). The longitudinal components are given as movements from west to east. They could equally well be regarded as movements in an east to west sense in which case the angles have to be subtracted from 360°. The latitudinal components are all northerly. Figure 9 also suggests that there has been some rotation of each land-mass and these amounts could be calculated. In all cases the longitudinal components are dominant and only in India is a latitudinal movement appreciable. The situation is similar in the Northern Hemisphere where the palaeomagnetic results suggest relative longitudinal movement of Europe and North America (Creer & others 1957). In reconstructions which assume Australia and the Australian poles to remain fixed, the longitudinal translations for the southern continents would be very much less than those given in Figure 9, but those for the northern continents become proportionately greater, being of the order of 15 000 km. In short, with the exception of India, the variations of geomagnetic latitude observed in different continents can be explained by polar wandering, and continental drift has, for the most part, succeeded only in changing their relative longitudes. Clearly if this statement is borne out by future palaeomagnetic observations (and very many more will be necessary) it will be the first and most important feature to be incorporated into any dynamical theory of continental drift.

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Caustics Produced by Waves Through the Earth's Core*

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Summary

Caustics produced by *SKP*, *PKS*, *PKP* and other phases through the Earth's core are investigated. No impulses from diffracted waves with periods of two seconds or more have been found beyond five degrees from caustics. Short waves preceding the main *PKIKP*-phase at distances between 125° and 140° probably have their deepest point in or near a transition zone between the liquid outer and the probably solid inner core and it is unlikely that they are related to the caustic of *PKP* near 145° although their travel-time curves end near this caustic. The observed range of long-period waves diffracted beyond the caustics of several phases is smaller than the maximum range calculated by Jeffreys.

1. The problems

Caustics and cusps of travel-time curves of waves travelling through the Earth's core are produced by two different phenomena. (1) The sudden decrease in the velocity of longitudinal waves at the core boundary results in travel-time curves consisting of two separate branches (Figure 1 a). In the following discussions

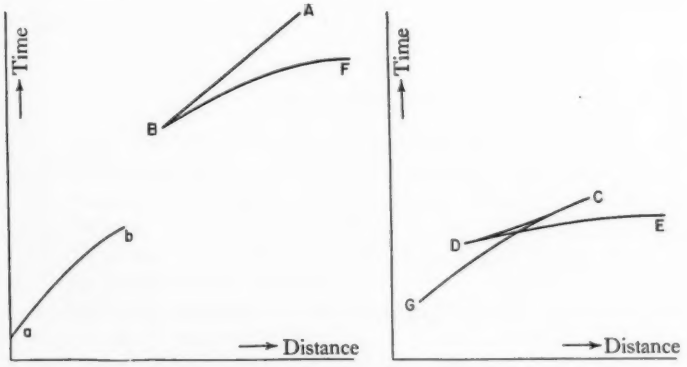

FIG. 1.—Schematic travel-time curves, (a) if the velocity decreases, (b) if it increases suddenly or rapidly with depth.

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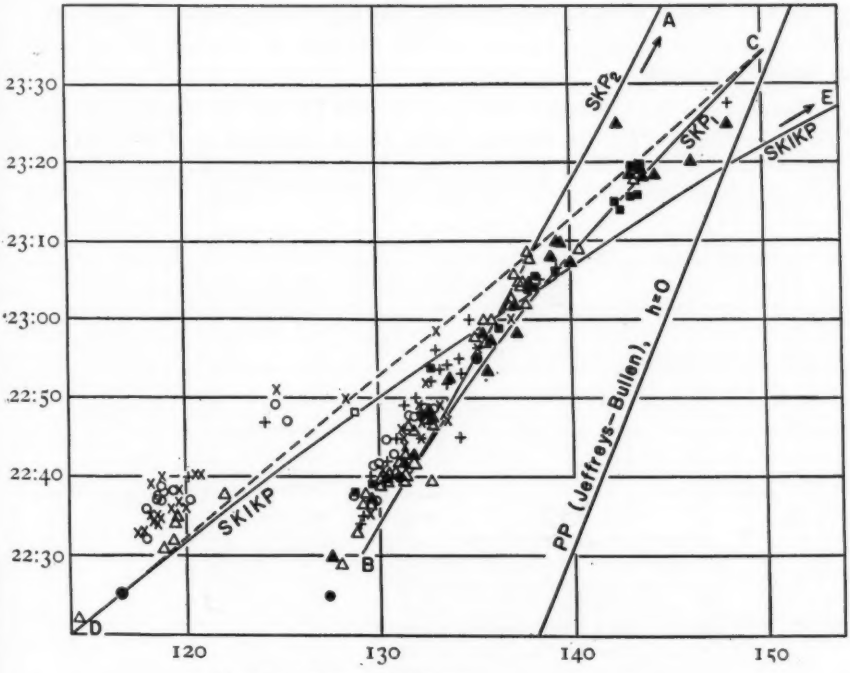
only the second branches (*ABF*) are involved. With decreasing angle of incidence the epicentral distance at which the rays arrive at the Earth's surface decreases to a minimum (at *B*), which corresponds to a caustic, and then increases again. (2) The increase in velocity between the outer and the inner core results in cusps of the travel-time curves (Figure 1 b). Most travel-time curves of waves through the core begin with a section *ABC* (where *C* is on the curve *BF* of Figure 1 a), then continue with *CDE* of Figure 1 b. For some phases, e.g. *PKS* and *SKP* (Figure 2), the



Abscissa: distance in degrees.
Ordinate: Travel time in minutes and seconds.

Observed data, corrected for h		Forester (1953) h = shallow,	
Data from SKP		short-period Z	△
Gutenberg (1958) h = shallow	+	long-period Z	▲
h = 50 to 150 km	×	h = 100 km, short-period Z	○
h = 270 km, Sweden	✳	Data from PKS intermediate h	●
h = 600 km	■	Curves are calulated for h = 0	

FIG. 2.—Travel times of *SKP* group, reduced to zero focal depth.

waves arrive noticeably earlier at the caustic *B* than waves to the branch *DE* at the same epicentral distance; for others, e.g. *PKP* (Figure 3), the caustic *B* is near the branch *DE*.

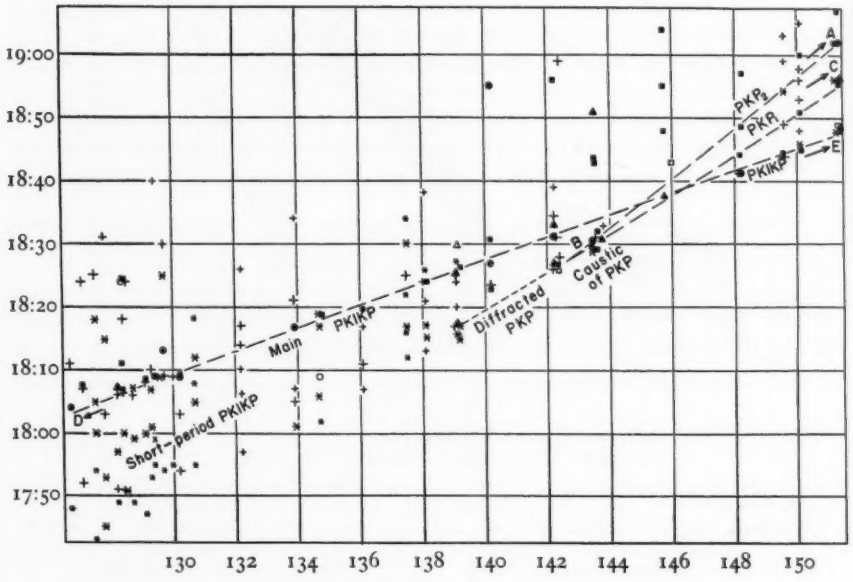
Wiechert (1922) has suggested that longitudinal waves through the core arriving at epicentral distances smaller than that of the caustic *B* which is at about 145° are diffracted, but Lehmann (1936) has pointed out that their amplitudes at distances as short as 110° are too large for diffracted waves. She suggested the existence of an inner core in which the velocity is greater than in the outer core. Gutenberg & Richter (1938) have found that assumption of an increase in velocity

from about 10·2 to 11·4 km/s between about 1600 and 1200 km from the Earth's centre explains the various observed travel-time curves of waves through the core.

Jeffreys (1939, p. 553) applying Airy's theory of diffraction has concluded that waves with periods T of 1 s could be diffracted up to 4 deg, and those with $T = 10$ s up to 15 deg from the caustic. To check these results, Bullen & Burke-Gaffney (1958) studied records produced by hydrogen bombs exploded near Bikini and written at epicentral distances slightly smaller than that of the caustic of *PKP*. However, the phases believed by Bullen and Burke-Gaffney to be diffracted *PKP* waves may actually be early short-period *PKIKP* waves (compare Figure 3).

2. Materials used

The material used includes data for *PKP* and *PKIKP* by Gutenberg (1951, 1957 a, 1958), for *PKP* by Denson (1950, 1952), for *PKS* by Forester (1953, 1956) and for *SKS* by Nelson (1952, 1954). Moreover, seismograms of an earthquake at a focal depth $h = 600$ km, have been kindly furnished by stations with



Abscissa: distance in degrees.
Ordinate: Travel time in minutes and seconds.

Travel times T	e	ei,i
< 1°	•	*
1 – 1¾	+	+
2 – 2¾	○	●
3 – 4¾	□	■
⩾ 5	△	▲

FIG. 3.—Travel times of the *PKP*-group, observed for the earthquake at 4 h: 04 min: 04 s G.C.T. on 1957 April 16, focal depth about 600 km near 4½° S 107½° E.

epicentral distances of over 119° in North and Central America (details in Table 1); for most of them the range of azimuths at the source is within 50°, so that the error in location (probably less than 1°) does not affect noticeably the resulting relative travel times.

Travel times t for *PKP*, *SKP* etc. to distances θ in degrees have been calculated by combining the travel times for K through the core found by Gutenberg (1958, Table 4) with the travel times for *PcP*, *PcS* and *ScS* for zero focal depth listed by Jeffreys & Bullen (1940). If $dt/d\theta$ for K and for *PcP* etc. is measured in seconds per degree, corresponding values of θ and t for *PKP* etc. are found by adding values for θ and t for K to those for *PcP* etc. which correspond to the same value

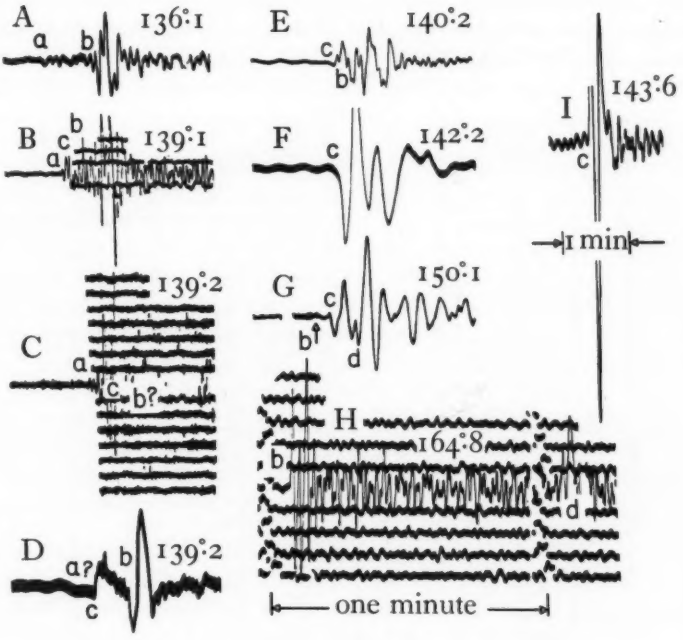


FIG. 4.—Beginning of vertical records of the shock of 1957 April 16 (Table 1). A. at Kirkland Lake; B. at Montreal; C. and D. at Ottawa, C from short-period, D from long-period instrument; E. at Lubbock; F. at Florissant; G. at Tacubaya; H. at San Juan; I. from ultra-long instrument at Palisades.

a. indicates short-period *PKIKP*; b. long-period *PKIKP*; c. *PKP*1; d. *PKP*2.

of $dt/d\theta$. This procedure permits rapid calculation and is sufficiently accurate in most instances. For *PKIKP*, travel times through the core for short as well as for long waves have been used, for all other phases only the main travel-time curve for K ($T \geqslant 2$ s).

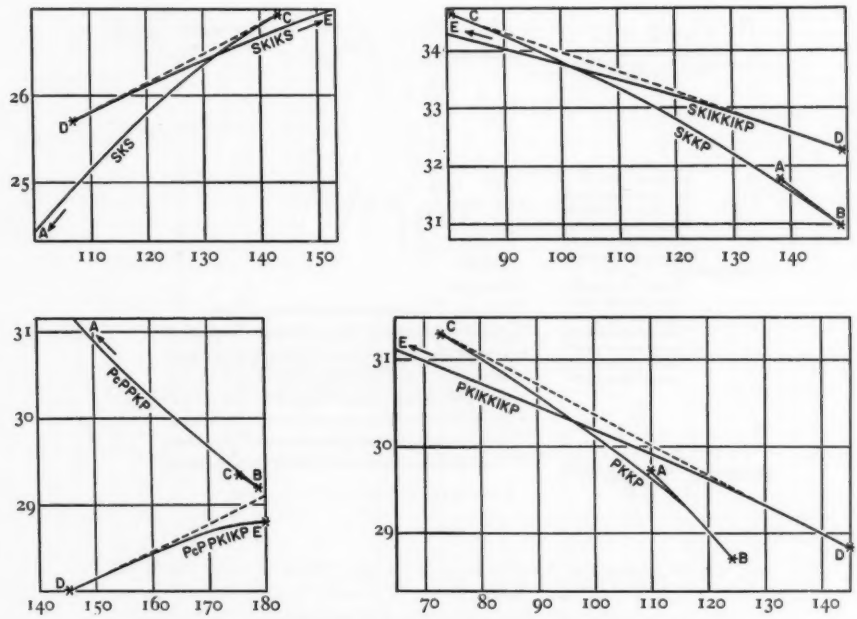
Where observed travel times have been reduced to values for a surface focus, the necessary corrections have been taken from Jeffreys & Bullen (1940) or from

Gutenberg & Richter (1936; 1939, p. 113). For *PKIKP* characteristic corrections are as follows:

Focal depth	25	100	300	600 km
correction $\theta = 145°$	+4	+14	+37	+69 s
correction $\theta = 180°$	+4	+14	+38	+70 s

For *PKP* and *PKS* they are about 2 s smaller than for *PKIKP* if $h = 600$ km. For *SKS* they are 125 s for $h = 600$ km at $\theta = 130°$ and 129 s near 180°; for *SKP*, most corrections are up to 2 s smaller than those for *SKS*. Errors in the corrections probably do not exceed a very few seconds.

The epicentral distance of the caustic of *PKP* is about 2° shorter for shocks at a depth of 600 km than for shallow shocks. However, while depth corrections for travel times depend mainly on the phase (*P* or *S*) with which the wave starts, the path in the core plays an additional role for the distance of the caustic.



Abscissa: for all diagrams. Distance in degrees.
Ordinate: for all diagrams. Travel time in minutes.

FIG. 5.—Calculated travel times of (a) *SKS*-, (b) *SKKP*-, (c) *PcPPKP*- and (d) *PKKP*-groups for zero focal depth.

3. SKP and PKS

In Figures 2, 3 and 5, *ABC* corresponds to waves which have not entered the inner core, the caustic is at *B*, and *CD* results from the increase in wave velocity between outer and inner core. Its proximity to *BC* and *CE* prevents observation, and its details are always uncertain. *DE* corresponds to waves refracted through the inner core. At *E*, which for *SKP* is at $\theta = 180°$, the amplitudes of *SKP* or

PKS are zero, since for vertical incidence at the core boundary no energy can be refracted from *S* to *K* or vice versa.

Most observed points in Figure 2 are either close to *SKP* near the caustic, or to *SKIKP* at shorter distances. Most observations of *SKIKP* are from records of earthquakes at intermediate depth (compare Gutenberg 1958, Figure 5). Observed times are frequently a few seconds greater than the calculated, probably mainly as a result of too late readings of phases emerging in the preceding motion.

For $h = 0$, the calculated epicentral distance of the caustic of *SKP* is 129°. Forester (1953, 1956) has found the largest amplitudes of *SKP* at distances near 131½° for waves with $T < 5$ s and between 130° and 131° if $T > 5$ s. The smallest distances at which he has identified *SKP* are 129° for short-period and 125° for long-period waves. The shortest distance at which *SKP* has been measured in the present research without searching for it is 129°; one clear *PKS*-wave ($T = 7$ s) has been found by Gutenberg (1958) from a 270 km deep shock at $\theta = 127°$. Thus, the observed distances of diffracted *SKP*- and *PKS*-waves are well within the theoretical limits found by Jeffreys (1939).

4. The caustic of PKP

Gutenberg (1914) had found that on records of the long-period instruments used before 1910 the first *PKP*-waves emerge at about 141° and that their largest amplitudes (periods 4 to 12 s) occur near 145°. Denson (1952) observed that *PKP*-waves with periods *T* of 5 to 8 s show the caustic near 143° while for those with *T* about 2 s it is near 147°. According to Gutenberg & Richter (1935, p. 325) the largest amplitudes of *PKP* are between 143° and 147°. In the present investigation, the largest amplitudes of *PKP* are observed at distances of between 142° and 144°, (compare Figure 4, F, I) where at several stations the first impulse is so large that its turning point is not visible. This corresponds to about 145° in a shallow shock. Gutenberg (1951, p. 382) has observed the amplitude maximum for *PKPPKP* at a distance of 68°, corresponding to a distance of 146° for *PKP*. An epicentral distance of 145° for the caustic of *PKP* in shallow shocks is in good agreement with all these data.

5. Travel times of PKP and PKIKP

Seismograms of the shock of 1957 April 16, 4 h: 04 min: 04 s, in the East Indies show clear phases of waves through the core (compare Gutenberg, 1957 a, Figure 1; 1958, Figure 4), and have been selected for detailed study. Twenty reports of *pP–P* ($50° < \theta < 104°$) give focal depths between 570 and 640 km (average 590 ± 5 km). For calculations $h = 600$ km, given by the U.S. Coast and Geodetic Survey and by the U.S.S.R., has been used.

Figure 3 shows travel times of the *PKP*-group; arrival times of the main phases are listed in Table 1. Short- and long-period records give the same time for the first impulse of *PKP*, which is not preceded by any detectable emergent waves (Figure 4, F, I) and near its caustic arrives a few seconds earlier than *PKIKP* (Figure 3).

At Ottawa ($\theta = 139{\cdot}2°$) a long-period wave which precedes *PKIKP* by about 9 s (Figures 3, 4 D) is probably a diffracted *PKP* wave. There is no indication of diffracted long-period *PKP* waves for $\theta < 138°$, that is, over 5° from the caustic. Consequently, we cannot expect short-period diffracted *PKP* waves for $\theta < 140°$, and it is unlikely that the short-period waves preceding the main *PKIKP* phase at $\theta < 140°$ are connected with *PKP*.

Table 1

Arrival times t (*after* 4 *a.m.*) *and corresponding periods* T *of first PKIKP-waves* (a), *main PKIKP-waves* (b), *PKP*1 (c), *and PKP*2 (d) *of earthquake on* 1957 *April* 16; *assumed epicentre at* $4\frac{1}{2}°$ *S* $107\frac{1}{2}°$ *E, focal depth near* 600 *km, magnitude* $m = 7{\cdot}2$. θ, *epicentral distance. Parentheses indicate that the interpretation is doubtful, including effect of microseisms.*

Station	θ	a		b		c		d	
		t	T	t	T	t	T	t	T
	deg	min s	s	min s	s	min s	s	min s	s
Seattle	119·5	no		21 55!	1½				
Ukiah; Shasta	122·7	no		21 59*	?				
Hungry Horse	123·8	no		22 01*	?				
Berkeley	123·9	no		22 02!	1;5				
Santa Clara	124·3	no		22 02	5				
Lick	124·5	21 49	¾	22 03!	1½				
King Ranch	126·7	(21 52)	¾	22 08!	2				
Tinemaha	127·1	21 56	1	22 11!	1;3				
Isabella	127·5	21 58	¾	22 09!	¾				
Eureka, Nevada	127·5	21 47*	?	?	?				
Woody	127·8	21 49	¾	22 07!	1				
China Lake	128·1	21 53	¾	22 10!	1½				
Pasadena	128·4	21 55	¾	22 11!	1;4;8				
Dalton	128·7	21 53	¾	22 11!	¾				
Riverside	129·1	21 51	¾	22 12!	1½;3				
Big Bear	129·3	21 57	¾	22 14!	1				
Salt Lake City	129·4	(22 03)	1	22 13!	2				
Schefferville	129·6	21 59	½	22 13!	½				
Palomar	129·7	21 58	¾	22 13!	1				
Boulder City	130·0	21 59*	?	22 13*	?				
Barrett	130·2	21 58	1	22 13!	2;4				
Hayfield	130·7	21 59	¾	22 16!	¾				
Rapid City	132·2	22 01	1	22 18!	1½				
Boulder, Colo.	133·9	22 05	¾	22 21!	2				
Denver	134·2	22 06	¾	22 21	2½				
Tucson	134·7	22 06	¾	22 23!	4				
Kirkland Lake	136·1	22 11	1	22 24!	3				
Seven Falls	137·5	22 16	¾	22 29	1				
Shawinigan Falls	138·1	22 17	1	22 28!	1;3				
Montreal	139·1	22 20	½	22 29!	1½;3	22 24	1		
Ottawa	139·2	22 19	¾	22 30!	4½	22 21!	½;3;5		
Halifax	139·3	22 21*	?						
Lubbock	140·2			(22 31)	2?	(22 27)	3		
Florissant	142·2					22 31	5		
Harvard	142·2					22 30!	1		
Weston	142·3			22 35	1;2	22 31!	1;4		
Cleveland	142·3					22 31!	1?		
St. Louis	142·4					22 32!	1		
Mazatlan	142·4					22 30	3		
Fayetteville	143·5					22 33!	1;3		
Pennsylvania State	143·5					22 35	3;10		
Palisades	143·6					22 35!	1;10		
Pittsburgh	143·6					22 36!	?		

Table 1 (*continued*)

Station	θ	a			b			c			d		
		t		T	t		T	t		T	t		T
	deg	min	s	s	min	s	s	min	s	s	min	s	s
Fordham	143·8							22	36*	?			
Dallas	143·8							22	37!	1;2			
Washington, D.C.	145·7							22	42!	5			
Chapel Hill	148·2				22	45	2;4	22	48!	4	22	52!	3
Columbia	149·6				22	48!	1;3	22	53!	1	22	58	½
Tacubaya	150·1				22	49!	½;3	22	56!	1;3	23	02	1;5
Bermuda	151·3				22	52	¾	23	00!	½	23	06	2
Mérida	156·6				23	02	3				(23	38)!	3
Comitan	157·5				(22	59)	3						
Oaxaca	157·8				23	04	3						
Antigua	163·6				23	05!	¾				24	00	¾
San Juan	164·8				23	06	1				24	07	1½
St. Lucia	165·2				23	06!	1				24	07!	1
St. Vincent	165·9				23	06!	¾				24	10!	¾
Trinidad	167·5				23	10!	¾				24	19!	¾
Galerazamba	173·2				23	15*	?						
Chinchina	176·9				23	12!	3				25	02!	3
Bogota	178·4				23	12!	4						

* From reports.

At distances between about 148° and at least 152°, *PKIKP*, *PKP*1 and *PKP*2 can be identified (Figures 3, 4 G). At greater distances, up to nearly 180°, *PKIKP* and *PKP*2 are clearly recorded (Figure 4 H).

At epicentral distances of $124° < \theta < 140°$, many seismograms of short-period instruments begin earlier than those of long-period instruments (Figures 3, 4 A). Travel times of these short-period phases, and especially the earliest that can be measured on a record beyond doubt, depend on properties of the instrument, the shortest periods existing in the waves, effects of the ground under the station, properties of the microseisms, and possibly the wave path, including effects of the ellipticity of surfaces in the earth. The time interval between these short-period waves and the main branch of *PKIKP* decreases with increasing θ. Ottawa ($\theta = 139·2°$) seems to be the most distant station where these short-period waves are visibly recorded (Figure 4 C).

These short-period waves preceding the main *PKIKP*-phase have been reported repeatedly (e.g. by Gutenberg & Richter 1934; Denson 1952; Båth in bulletins of Uppsala). They have been considered by Gutenberg (1957, 1958) to be *PKIKP*-waves which have higher velocities in the transition zone from the outer to the inner core than the longer waves. Kuhn & Vielhauer (1953) have concluded from theoretical considerations and from laboratory experiments that in a material near its melting point the velocity of longitudinal waves depends on the wave length, and that for a given material the "boundary" between its solid and its liquid phase, found from observed increase in wave velocities, should extend farther towards the liquid phase for short than for relatively long waves. If we assume (probably not correctly) that all the earliest short "*PKIKP*"-waves belong to the same travel-time curve, an increase in velocity of longitudinal waves

from about 10 to 11 km/s would start at a distance r of about 1700 km from the Earth's centre; exact values of r cannot be found, since neither the epicentral distance of the end of *PKP* (point *C* in Figure 3) nor the branch *CD* of the travel-time curve is observed. For waves with travel-times about half way between the first and the main branch of *PKIKP* in Figure 3, Gutenberg (1958) found on similar assumptions that the longitudinal wave velocity would increase from about 10·1 km/s at r about 1500 km to 11·2 km/s at $r = 1300$ km; for the main branch (T usually 2–3 s) a similar increase in velocity would occur between 1300 and 1200 km from the Earth's centre.

The greatest difference in travel times for short- and long-period *PKIKP*-waves should occur at the shortest distances. With increasing θ the length of the path of short-period waves in the transition zone between outer and inner core decreases, and the various travel-time curves should approach each other as it is observed (Figure 3). Unfortunately, periods T of less than 1 s cannot be measured accurately on usual seismograms; theoretically, the effect of the transition zone should differ appreciably for waves having different periods of less than 1 s (length $10T$ km). Irregularities in the transition zone should produce greater scattering and higher absorption of short than of longer waves. The curves for the melting point and the temperature in the transition zone probably intersect at a very small angle, so that a thick transition zone with irregularities may be expected.

The rather sudden end of the short-period waves at about the same distance and time where the long-period diffracted *PKP* begins looks suspicious. However, it is unlikely that short-period diffracted *PKP*-waves extend many times as far from the caustic as long-period diffracted *PKP*-waves. Another problem concerns the short-period impulses observed frequently simultaneously with the main long-period *PKIKP*-impulse. However, the shortest periods observed in this impulse are usually noticeably longer than those observed in the earlier short-period waves (compare Table 1). The finding of several distinct impulses between the first and the main *PKIKP*-waves and the lack of a continuous spectrum corresponds to the observation (Gutenberg, 1957 b) that in many seismograms from distances of less than 100° *P* as well as *S* show the prevalence of a few distinct periods and not a continuous spectrum.

6. Travel-time curves of other phases through the core

Observed travel times of *SKS* follow the curve for *SKS* (Figure 5 a) at short epicentral distances as far as its intersection with the curve for *SKIKS* and are near the *SKIKS*-curve for greater distances (Nelson 1954).

Travel times for *PKPPKP* can be found from Figure 3 by doubling distances and times. Observations by Gutenberg (1951, Figure 3) are close to all major branches. There is no indication that *PKPPKP* is observed more than 8° beyond the caustic; this corresponds to 4° for *PKP*.

Figure 5 d shows travel-time curves for *PKKP*, which is a short-period phase. There are many observations by Gutenberg (1951, Figure 12), most between 2 s earlier and 12 s later than calculated. *PKIKKIKP* has not been definitely observed. There is no indication of *PKKP*-waves diffracted noticeably beyond the caustic.

Figure 5 b shows travel-time curves for *SKKP*, which usually consists of a train of short-period waves. Impulses of *SKKP* have been recorded at eleven stations ($129° < \theta < 144°$) on 1957 April 16. They are between 3 and 13 s later than calculated. *SKIKKIKP* has not been observed.

The travel-time curve of *PcPPKP* (or *PKPPcP*) should have a caustic near

180° (Figure 5 c). The curve for *PcPPKIKP* has a strong curvature near 175°, so that relatively large amplitudes can be expected there. Strong impulses have been observed by Gutenberg & Richter (1934, p. 123) at distances between 175° and 176° with travel times between 29·0 and 29·3 min.

Other phases through the core and their caustics have been discussed by Gutenberg & Richter (1934, 1939) and by Jeffreys & Bullen (1940).

7. Conclusions

In all instances where seismograms were available showing phases at distances near their caustic, or where corresponding data have been reported, no waves having periods $T \geqslant 2$ s have been found in the "shadow zone" at distances exceeding about 5 deg beyond the caustic. The amplitudes of these diffracted waves decrease rapidly with increasing distance from the caustic. The range of distances showing waves diffracted at the caustic surfaces in the Earth is even shorter than that expected theoretically by Jeffreys. It is unlikely that diffracted waves with periods of two seconds or less exist at epicentral distances exceeding five degrees from these caustics. Waves with periods of less than 2 s, observed at distances of between 19° and 4° from the caustic of *PKP* are probably *PKIKP* waves with their deepest point in or near the transition zone between the liquid outer and the solid inner core; in this zone, short waves travel faster than longer waves.

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Notes on Progress in Geophysics

Geophysics in South Africa

A. L. Hales

Summary

The scope of this review is more restricted than is suggested by the title for it deals only with those branches of fundamental geophysics which are concerned with the solid Earth.

Gravity

In so far as the study of the Earth's gravity field is concerned, 1958 is a year of considerable achievement in South Africa, for a few months ago the Geological Survey published a Bouguer anomaly map of South Africa as an overprint on the new edition of the 1 : 1M geological map of the Union of South Africa.

The publication of this map brings to completion a programme which began in 1948 with a series of pendulum measurements by Hales and Gough (1950). On the basis of the gravity stations established then, Officers of the Geological Survey* have built up a network of gravimeter stations covering the whole of the Union.

The most spectacular anomaly shown on this map is undoubtedly that centred on Trompsburg which was found by B. D. Maree in about 1942. A map showing this anomaly is reproduced as Figure 1.

In Part I of the memoir describing the gravimeter programme, Smit will discuss the Bouguer anomalies while in Part II Hales and Gough will deal with the isostatic anomalies. The isostatic reductions have been made on five hypotheses with regard to the distribution of the compensation, namely, the Airy-Heiskanen hypothesis with crustal thicknesses of 20, 30, 40 and 60 km and the Pratt-Hayford hypothesis with depth of compensation 114 km. In a preliminary analysis of the observations in 1954 the regression coefficients of anomaly on height given in Table 1 were found.

The limits quoted are the 99 per cent confidence limits. These coefficients suggest that the crustal thickness is greater than 60 km. It was pointed out that this was probably due to the tendency for there to be a belt of positive anomalies

Table 1

Hypothesis	Free Air	AH 20	AH 30	AH 40	AH 60	Bouguer
Regression	+0·0059	+0·0026	+0·0019	+0·0014	+0·0004	−0·0274
Coefficients	±0·0022	±0·0016	±0·0016	±0·0016	±0·0016	±0·0022

* The observation programme was planned by J. F. Enslin and L. E. Kent and carried out by P. J. Smit.

along the top of the escarpments with a belt of negative anomalies at the bottom. This kind of departure from exact compensation is that which would be expected if the compensation were regional as proposed by Vening Meinesz. Using the Bowie criterion that the best estimate of crustal thickness is given by the minimum sum of the squares of the departures from the mean anomaly, it was found that the

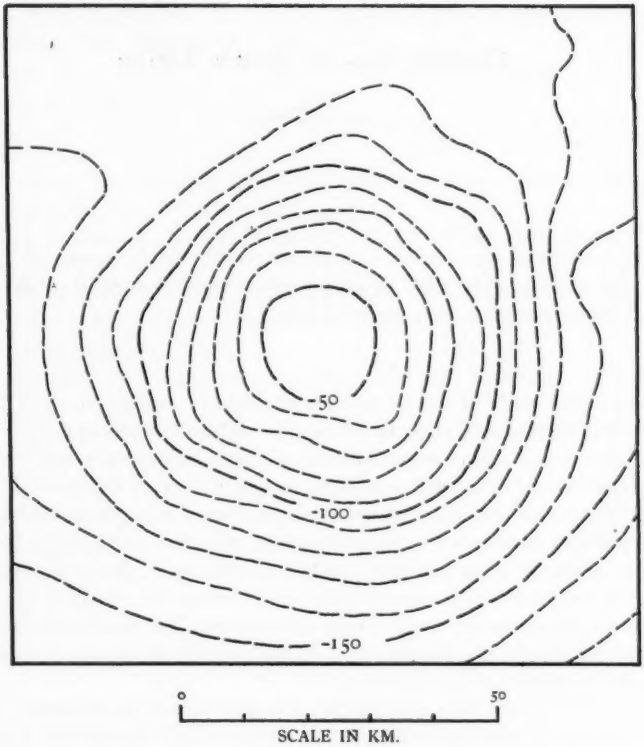

FIG. 1.—The Trompsburg gravity anomaly. Contours represent Bougner anomalies in milligals.

estimated crustal thickness was between 35 and 40 km, in reasonably good agreement with the estimates obtained from seismological studies.

During the past ten years pendulum and gravimeter measurements have been made by Rose, Harding and Bonini of Woollard's group at some South African stations. In 1956 Gough made a new connection to Teddington using the Cambridge pendulums, and recently Browne has made observations at a number of stations in Africa including Johannesburg. Harada, Suzuki and Ohashi made observations at Mowbray during the Japanese Antarctic Expedition. The observations so far known agree well with the 1948 determinations and it is clear that the South African stations are firmly tied to the international network.

Seismology

In a paper published in 1946 (Gane, Hales & Oliver 1946) it was shown that the energy of Witwatersrand earth tremors was of the order of 10^{16} to 10^{19} erg.

It was clear that these tremors could be recorded by suitable equipment at very great distances. Gane and his colleagues built up equipment for automatic recording of the tremors at the Bernard Price Institute and at field stations at distances of up to 400 miles from Johannesburg (Gane, Logie & Stephen 1949). The study of crustal structure by means of the travel times of the waves generated in these tremors began in 1948 with work in the Western Transvaal (Willmore, Hales & Gane 1952). Later Gane extended the investigation to the south, north and east of Johannesburg (Gane, Atkins, Sellschop & Seligman 1956). It was found that combining the results from all traverses the crustal velocities were 6·18 km/s for P and 3·66 km/s for S. The subcrustal velocities were 8·27 and 4·73 km/s respectively. The crustal thickness deduced from P observations was 35 km and from S observations 34 km.

It was clear at this stage that there were two aspects of crustal structure which had not been covered completely. In the first place it seemed desirable that an attempt should be made to see whether there were any changes in travel time associated with the changes in crustal thickness to be expected at an escarpment, and in the second, that investigations should be made of what limits could be placed on the velocity distribution within the crust. The importance of the second question was stressed by Birch.

For this purpose Sacks designed and built new transistorized field equipment, some of which has been described (Sacks 1957). Observations were made at various times during 1956 and 1957 by Hales and Sacks, and the results are now being analysed. Careful observations in the neighbourhood of the distance at which P_n overtakes P_1 established that there was no first arrival phase other than these two. On this basis it was possible to set limits on the thickness of the "intermediate layer" for any postulated velocity in that layer and to show that the error in crustal thickness which could arise from interpreting the travel times in terms of single layer theory could not exceed 5 km.

It was found later that there was evidence of a P phase with velocity of about 7 km/s. It was found too, that the distance at which S_n overtakes S_1 is considerably greater than that at which P_n overtakes P_1 so that there would be a discrepancy in the estimates of crustal thickness from the P and S travel times unless there is an "intermediate layer" in which the S velocity increases proportionately rather less than P as was suggested earlier (Hales 1951).

Until recently no seismic refraction work at sea had been done in South African waters, but during April, 1958, the Lamont Geological Observatory ship *Vema* working in co-operation with *Vrystaat*, a South African naval vessel, made five reverse traverses off the South African coast. It is hoped that it will be possible to do more of this work in future.

In the field of earthquake seismology the Bernard Price Institute has for more than ten years operated four stations equipped with Benioff short-period vertical seismographs on behalf of the Geological Survey of the Union. In addition, there are two Milne Shaw seismographs at the Magnetic Observatory, Hermanus, and for the International Geophysical Year an additional station has been set up at Windhoek. During the I G Y also the three stations, Hermanus, Grahamstown and Pietermaritzburg, which are equipped with long-period instruments, have made 6-hourly readings of microseism level and period. Monthly bulletins of the observations are sent to B C I S at Strasbourg. The operation of these seismographs has shown that there are many more small earthquakes in the South African area than had been expected. In a recent paper H. O. Oliver (1956) has summarized the observations of the past ten years.

Terrestrial Magnetism

The South African centre for measurements of the present magnetic field of the Earth is the Magnetic Observatory, Hermanus. The ordinary observing programme of the Observatory has been extended for the International Geophysical Year to include cosmic ray observations and measurements of earth currents as well as additional variograph recording. Of greatest importance from the point of view of conditions inside the Earth are the secular variation surveys at 53 stations in the Union and Rhodesia which the Observatory makes at five or ten year intervals (van Wijk 1953, van Wijk & Viljoen 1953). van Wijk (1954) has drawn attention to an annual variation in the records from Hermanus. This effect might be important in determining the electrical conductivity at great depths in the mantle.

The first paleomagnetic measurements on South African rocks were made by Gough. The rocks studied were dykes of the Pilanesberg system (Gelletich 1937, Gough 1956). The first samples measured were drilled from surface outcrops and showed almost random scatter. Samples from underground workings in the mining areas showed extremely good consistency, the mean direction of magnetization corresponding to a north pole in latitude $7\frac{1}{2}°$ N, longitude $42\frac{1}{2}°$ E. The pattern of inconsistent results from surface samples has been repeated throughout the South African observations. Gough has studied samples collected by van Niekerk from 57 flows in the Stormberg lavas. The results were not random but fell into three or four groups and did not provide satisfactory data for paleomagnetic purposes. Graham and Hales (1957) found large scatter in samples of Karroo dolerites collected from many parts of the Union. However, samples collected underground from two areas 200 miles apart again showed consistency. Hales and Graham have measured samples covering virtually the whole sedimentary succession of the Karroo but it is not possible to see any consistent pattern in the results. Measurements on the lower red shales of the Table Mountain series showed the closest approach to consistency but even here there were two possible directions.

In an effort to find why the South African observations showed so much more scatter than those obtained elsewhere drilling equipment has been built with which it is possible to obtain accurately oriented cores to a depth of 6 feet. Detailed sampling has been carried out near one of Gough's surface sampling sites. The variation with depth and in the horizontal plane are consistent with the view that the scatter found by Gough is due to magnetization by lightning. At a depth of 6 feet the direction of magnetization approaches closely to that found by Gough from the underground samples. The depth to which magnetization by lightning is effective is of course dependent on the magnetic properties of the rock and it is hoped that by considering these properties it will be possible to find places where satisfactory paleomagnetic data can be obtained within 10 or 20 feet of the surface. Gough and van Niekerk are at present making measurements on the 2 000 million year old norite from the Bushveld Complex. First results suggest that the magnetization is stable.

The age of rocks

The possibility of age measurements based on the decay of rubidium-87 to strontium was pointed out by Hahn & Walling (1938), and in 1947 Ahrens (1947) published the first series of measurements by this method. Ahrens determined the concentration of rubidium and strontium by spectrochemical methods but it

was later shown by Davis and Aldrich that greater accuracy could be obtained by using the isotope dilution method.

At the present time there are two active groups doing age determinations in South Africa. At the National Physical Research Laboratory the mass spectrometry division makes measurements by the various methods which are applicable to uranium or thorium minerals. At the Bernard Price Institute of Geophysical Research a unit which was started with the help of the Nuffield Foundation and the Carnegie Corporation of New York makes determinations by the rubidium–strontium method.

The first work done at NPRL was the determination of the ages of uraninites from many different parts of the Witwatersrand system by Louw (1954). Louw found that there was a fairly strong concentration of the ages of the uraninites between 1 850 and 1 950 million years. If one accepted the view generally held in South Africa at that time that the uraninites were, in the main, detrital, then Louw's results could be interpreted as giving the age of the material from which the rocks of the Witwatersrand were derived. It followed that the Witwatersrand system was younger than 1 950 million years. Later work by Liebenberg (1955) has shown that although the uraninite is largely detrital there has been secondary recrystallization. Thus Louw's measurements do not give an upper limit of age for the Witwatersrand system. Instead, there is now a lower limit of age, for Mendelssohn, Burger, Nicolaysen & de Villiers (1958) have reported an age of 2 180 ± 30 million years for a monazite described by Mendelssohn and Marland (1933) which is authigenic in origin and therefore younger than the Witwatersrand system.

The first publications from the Bernard Price Institute dealt with ages of lepidolites (Schreiner, Jamieson & Schonland 1955, Jamieson & Schreiner 1957) and the isotope dilution technique (Jamieson & Schreiner 1956). It was then decided that it would be more satisfactory to concentrate on methods for determining the ages of the granites themselves for there was always the possibility that determinations on pegmatites might not give the true age of the granite in which they were found. Schreiner concentrated on measurements on a solution of the whole of the rock sample which he has called the "total rock" method. He also made measurements on the light or felspar fraction and on biotite. The total rock method was devised because of doubts about the role which might be played by diffusion of rubidium and strontium in the rock. It is clear that measurements of the age by the total rock method would not be affected by diffusion from mineral to mineral within the rock. Furthermore if the ages determined from the total rock, the light fraction and the biotite agree, then it is very improbable that the ages found have been affected by any kind of geological accident in the history of the rock for it is unlikely that the concentrations of rubidium and strontium in the different fractions would have been altered in exactly the right proportions to give consistent ages.

The first granite investigated was the red granite of the Bushveld Igneous Complex. For this granite Schreiner (1958 a) found that the total rock ages of four samples from widely separated parts of the Complex had a mean age of 1 840 ± 160 million years while the felspar fraction of the same sample gave an age of 1 990 ± 160 million years. (The limits quoted are 99 per cent confidence limits and the half-life of rubidium-87 was taken to be $5{\cdot}0 \times 10^{10}$ years.)

Nicolaysen, de Villiers, Burger & Strelow (1958) reported measurements on zircons from one of the samples and on the biotites from three of the samples.

The ages found for the biotites were 1 950 ± 50, 1 890 ± 60 and 1 690 ± 90 million years, the limits quoted being standard errors. In the case of the zircons, measurements were made on samples of different mesh sizes and the ages inferred were consistent with those found for the biotites. The same authors report measurements on a number of samples of monazite from Houtenbek in the Complex and give a mean age of 2 010 ± 80 million years. Finally, they report an age of 2 050 ± 50 million years on a biotite bearing pyroxenite from the Rustenburg Platinum mine.

Cooke (1953) in reviewing South African stratigraphy takes the age of the Bushveld Igneous Complex to be about 650 million years. The great weight of experimental evidence discussed in the preceding paragraphs makes it clear that the Bushveld Igneous Complex is, in fact, very much older and this fact will have important consequences for South African stratigraphy.

Age determinations had been made on a number of pegmatites cutting the granite in the North-West Cape (Nicolaysen 1954, Tilton & Nicolaysen 1957) and these clustered round 1 000 million years. It was inferred that the granite-gneiss in that region was of the same age. However, in the Prieska region the granite-gneiss is overlain by the Transvaal System which is older than the Bushveld system. There is, therefore, an inconsistency.

In an effort to resolve this inconsistency Nicolaysen collected samples of a pegmatitic granite from within two miles of the granite-Transvaal system contact. The age of this was found to be 1 090 million years so that the inconsistency was not resolved. At a later stage a sample of granite was collected within a few feet of the contact and was found to have an age of 2 100 million years. Clearly, the relations between the pegmatites and granites in this area must be investigated in much more detail both in the field and in the laboratory.

One of the difficulties in paleomagnetism lies in ensuring that comparisons of the direction of magnetization of rocks in different continents are being made between samples of the same age. This difficulty arose in particularly acute form in connection with the measurements made by Gough on Pilanesberg dykes. Fortunately it has been possible to resolve the uncertainty in this case. van Niekerk* studied the petrology of the rocks and found that some samples contained a small amount of biotite. By careful concentration Schreiner and van Niekerk were able to secure samples with a sufficiently high proportion of biotite for measurement of the age which was found to be 1 290 million years (Schreiner 1958 b; Schreiner & van Niekerk 1958). In this case the rubidium–strontium ratio was too low for measurements on the total rock but the three samples on which measurements were made had considerably different ratios of rubidium to strontium and a similar argument to that given for the Bushveld granite can be used to show that it is unlikely that the ages have been affected by any "accident" in their subsequent history.

At present work is proceeding on rocks from the Eastern Transvaal and Swaziland, and on galenas from the Witwatersrand system, while Burger is investigating the discordant ages found in monazites.

Attempts have been made elsewhere to find a method for determining the age of sediments. It would seem that direct determination of the age of sediments will only be possible in exceptional cases. Direct determination of the ages of the interbedded lavas would, however, be equally effective in dating sedimentary systems. Consideration of the analyses of rhyolites suggested to the author that these lavas often had high ratios of potassium to calcium and that therefore it

* For an M.Sc. thesis at the University of Pretoria.

should be possible to make age measurements on these lavas by the total rock method. This has proved to be the case for two ancient lavas.

Andesitic lavas sometimes contain biotite mica and if the problem of separation of the mica could be overcome, even if only incompletely as in the case of the Pilanesberg, then these lavas could also be used for age determination.

It is the view of South African workers that age determinations can play an important role in problems of stratigraphy, but it is of course necessary that the samples should be collected from places where the field relations are well established. Dr F. C. Truter, Director, and Dr O. R. van Eeden, Deputy-Director, of the Geological Survey of the Union and Dr H. J. R. Way, former Director, and Mr D. R. Hunter of the Swaziland Survey have been very helpful in giving information on field relations and in the collection of samples.

Heat flow

The pioneer work in the measurement of heat flow in South Africa was done by Bullard (1939) using temperature measurements by Krige (1939). More recently measurements in the Transvaal and Orange Free State have been reported by Carte (1954). The heat flows found ranged from 0·92 to $1{\cdot}37 \times 10^{-6}$ cal cm^{-2} s^{-1}. Gough has measured the temperatures in a group of deep boreholes in the Southern Cape, and measurements of the conductivities are to be made shortly.

International Geophysical Year

A great deal of geophysical work is being done in South Africa as part of the International Geophysical Year Programme. However, most of this work lies in the fields of meteorology, ionospheric physics, oceanography and other disciplines which are outside the scope of this review. The South African National Committee has published a pamphlet which gives a comprehensive picture of the South African International Geophysical Year programme.

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Geophysics, Pure and Applied*

C. L. Pekeris

Geophysics is the science of the physical processes in the Earth. Nowadays, geophysics embraces the study of the internal constitution of the Earth, of the oceans, the atmosphere and the outer space beyond. The territory of geophysics extends thus from the Sun to the centre of the Earth.

I wish to give you a short review of some geophysical problems which have been investigated by the mathematicians of the Weizmann Institute. I shall take you on a sightseeing tour, starting from the centre of the Earth and progressing from there outward.

And so we ask, what goes on at the centre of the Earth? Do we know, for instance, the magnitude of the pressure there, of the temperature, the force of gravity, the nature of the material?

It is clear that we do not have any direct information on these questions. About the only source of information that we possess on the internal constitution of the earth is earthquake records. The wave produced by an earthquake penetrates deep into the interior of the Earth and returns again to the surface laden with information on what it saw on its journey. Unfortunately, this story is spelled out in a code which can be properly deciphered only with the aid of the most subtle tools available to the applied mathematician, including, of late, electronic computers. One interesting result that had already emerged from these seismological studies before World War I is that the Earth is made of solid rocks only down to a depth of 2 900 km. From there on down to the centre of the Earth, that is over a distance of 3 500 km, the material is *liquid*. Hence the term: the *liquid core* of the Earth.

Let us see now what we can say about the state of affairs at the centre of the Earth. Ladies and Gentlemen, I doubt whether you will learn anything of practical use from my talk. But if some of you are interested in losing excess weight, I can give you a prescription which is guaranteed to work. He who succeeds in reaching the centre of the Earth will lose, not only his excess weight, but *all* of his weight. The reason for this is that the force of gravity is exactly zero at the centre of the Earth. This is the one bit of information about the state at the centre of the Earth of which we are absolutely certain.

We are also reasonably sure that the *pressure* at the centre of the Earth is from three to four million atmospheres. This means that, on the average, the pressure increases at the rate of from 500 to 600 atmospheres for each increase of a kilometre in depth. At the bottom of the Helez oil wells, the pressure is about 300 atmospheres. Since these wells are a kilometre and a half deep, it follows that near the surface the pressure increases at the rate of only about 200 atmospheres per kilometre. The reason for the difference is that the density of the Earth's material increases with depth. Whereas the density of the rocks near the surface is about

* Address delivered on 1958 May 6, on the occasion of the award of the Weizmann Prize in the Exact Sciences by the municipality of Tel-Aviv.

2 g/cm^3, the average density of the material of the Earth is 5·5 g/cm^3. At the very centre of the Earth the material may be as much as nine times heavier than surface rocks, as we shall see later.

As for the temperature at the centre of the Earth, it is best that we admit our ingorance at the outset. It is generally believed that it is around 10 000° C, but this value depends on the assumptions one makes about the distribution of radioactive materials inside the earth.

What is the density at the centre of the Earth? If we knew the density of the material at the centre, we might be able to identify it. The density of iron is between 7 and 8, that of lead 11, uranium 19, platinum 21. These values hold under atmospheric pressure. Under a pressure of 3 to 4 million atmospheres, the densities are likely to be about twice as large.

There exist nowadays two views as to the value of the density of the centre of the Earth. According to a model of the Earth proposed by the Australian geophysicist Bullen (1950), the density at the centre of the Earth is around 18, whereas Sir Edward Bullard of Cambridge has proposed a model in which the central density is only about 13 (Bullard 1957).

Our research on the internal constitution of the Earth has recently brought out certain theoretical results which tend to favour Bullen's model over Bullard's. These investigations were concerned with the interpretation of the seismograms of an earthquake which occurred in Kamchatka on 1952 November 4, five days before the death of President Chaim Weizmann. Professor Benioff of Pasadena, California, discovered on his records of the Kamchatka earthquake two peculiar oscillations, one of a period of 57 minutes and another one of a period of about 100 minutes (Benioff 1954). Normally when an earthquake wave passes a station, the ground vibrates at the rate of about ten to one hundred times *each second*. Sometimes one also finds oscillations in which one swing lasts about a minute. In the case of the Kamchatka earthquake Benioff for the first time recorded oscillations of a period of the order of a whole hour.

Benioff suggested that these long period oscillations might represent the natural tone of the Earth. If you compare the tone of bells of different sizes you will find that the larger the bell the lower is its pitch. If you make the size of the bell unwieldy, the pitch will become so low as to be inaudible. When the bell grows to the dimensions of the Earth it will move only one swing in one hour, instead of more than a thousand swings in a *second*, which a house bell makes.

The purpose of our investigations, which were carried out on the electronic computer of the Weizmann Institute, was first to test the validity of Benioff's hypothesis, and secondly to see whether the observed periods are sensitive to the internal constitution of the Earth. The first result which Mr Jarosch and I obtained was that if we assume the Earth to be solid from the surface down to the centre, the period of free oscillation turns out to be only three quarters of an hour, rather than the observed value of close to a whole hour (Pekeris & Jarosch 1958). This result lends additional support to the now accepted view of the existence of a liquid core in the Earth. Subsequent investigations, in which Dr Z. Alterman joined us, showed that if we do assume the existence of a liquid core, then *both* Bullen's and Bullard's models give a period of free oscillation of about 54 min, which agrees within the experimental error with the observed value of 57 min. The 57 min oscillation therefore provides no criterion for choosing between the two models.

However, when we come to the slower oscillation, we find that Bullen's model

has another oscillation of a period of 101 min, whereas Bullard's model has no free oscillation anywhere around 100 min. This theoretical result therefore provides evidence favouring Bullen's model of the Earth over Bullard's, since Benioff did observe an oscillation of a period of about 100 min in the Kamchatka earthquake.

Tentative though this result may be, and there exist at the moment certain theoretical difficulties which yet need to be overcome, it is of interest to inquire what could the material at the centre of the Earth be if its density were 18 rather than 13? If the liquid core consisted of molten iron, as has been proposed by some geophysicists, then a value of the density of 13 would fit iron under a pressure of the order of 3 to 4 million atmospheres. However, with a value of the density at the centre of 18, even the proferritic school of geophysicists would have to admit that the material at the centre of the Earth is different from molten iron. Precisely what it is one cannot tell at this stage. So much for the centre of the Earth.

My limited time does not allow me to dwell on some very interesting phenomena that one encounters on passing between the centre and the outer crust of the Earth, which constitutes the top layer of about 40 km in thickness. I shall only point out that one of the current problems in pure geophysics is the measurement of the thickness of the crust over the globe. This thickness varies, being greater under mountain ranges, for instance, and less under the oceans. This problem is being actively pursued principally in Russia and in the United States. The Russians are using the methods developed by the late academician Gamburtsev (Gamburtsev 1952, Gamburtsev, Weizmann & Tulina 1958). It may interest you to know that Gamburtsev's work is being carried on by his school, among whom is his wife Perl Weizmann (Weizmann, Kosminskaya & Riznichenko 1957), a niece of Chaim Weizmann. In the United States, some very interesting advances have been recently made by Drs Tatel and Tuve (1955, 1957). The latter obtained for the first time a direct echo from the bottom of the crust.

A peculiar feature of this echo is that it is heard only in the range of 80 to 90 km from the explosion. Why 80 km?

Independently of Tatel and Tuve's observations, Dr Longman, H. Lifson and I (Pekeris & Longman 1958, Pekeris & Lifson 1957) encountered a similar phenomenon in our studies of "theoretical earthquakes". These are models of earthquakes formulated purely mathematically and registered, not on a seismograph, but on WEIZAC, our electronic computer. Our goal in these mathematical studies was to ascertain what a theoretical earthquake record should look like. There too we found that at a distance from the explosion corresponding roughly to 80 km in the case of the Earth's crust, the character of the seismogram changes radically, and a sharp pulse suddenly sets in. Our theoretical studies thus throw light on the nature of the echo from the crust which Tatel and Tuve observed.

We now come to depths below the surface of the Earth of about a kilometre or two, which are of interest for oil prospecting. As you may have heard, the Department of Applied Mathematics of the Weizmann Institute has carried out nearly all of the geophysical prospecting for oil in Israel. We used two methods—the gravimetric and the seismic. If I had hung a pendulum in this hall before your arrival, and compared its direction after the hall was filled, I would have found that the pendulum swung towards you. This is because of the attraction which the audience exerts on the pendulum. Similarly, if heavy material is buried underground, its presence can be detected on the surface by the extra gravitational pull it exerts. By using very sensitive instruments we have measured the force of gravity

over all of those regions in Israel which are of interest for oil prospecting. In this way we obtained a *gravity map* of Israel, a portion of which is shown herewith (Figure 1). You will note that the force of gravity increases as you go from the coastal plain eastward towards the Judean hills. The reason for this is that the Judean limestones, which you see exposed on the road to Jerusalem, are heavier than the sediments covering the coastal plain. To my knowledge, Israel is the only country in the world where a complete gravitational survey was completed with the precision required for oil prospecting. The gravitational survey was headed by Dr J. Gillis, and later by Mr A. Ginzburg. A total area of close to 8000 km^2 has been mapped gravitationally.

In the seismic method of geophysical prospecting we produce an artificial earthquake and record the echoes that bounce back from the various geological strata at depth. A total of a thousand kilometres of seismic profiles have been made so far. This work was headed by Dr S. Meiboom and later by Mr M. Behr.

I shall add that though only about ten per cent of the oil consumption has so far been discovered, the prospects of further discoveries are good, provided there be no let-up in exploration.

When we come to sea level, we are faced with another mathematical problem in the shape of the tides in the ocean. In 1775 the great French mathematician Laplace (1775) produced a theory according to which one can predict the height of the tide at any coastal station in the world, provided two conditions are met:

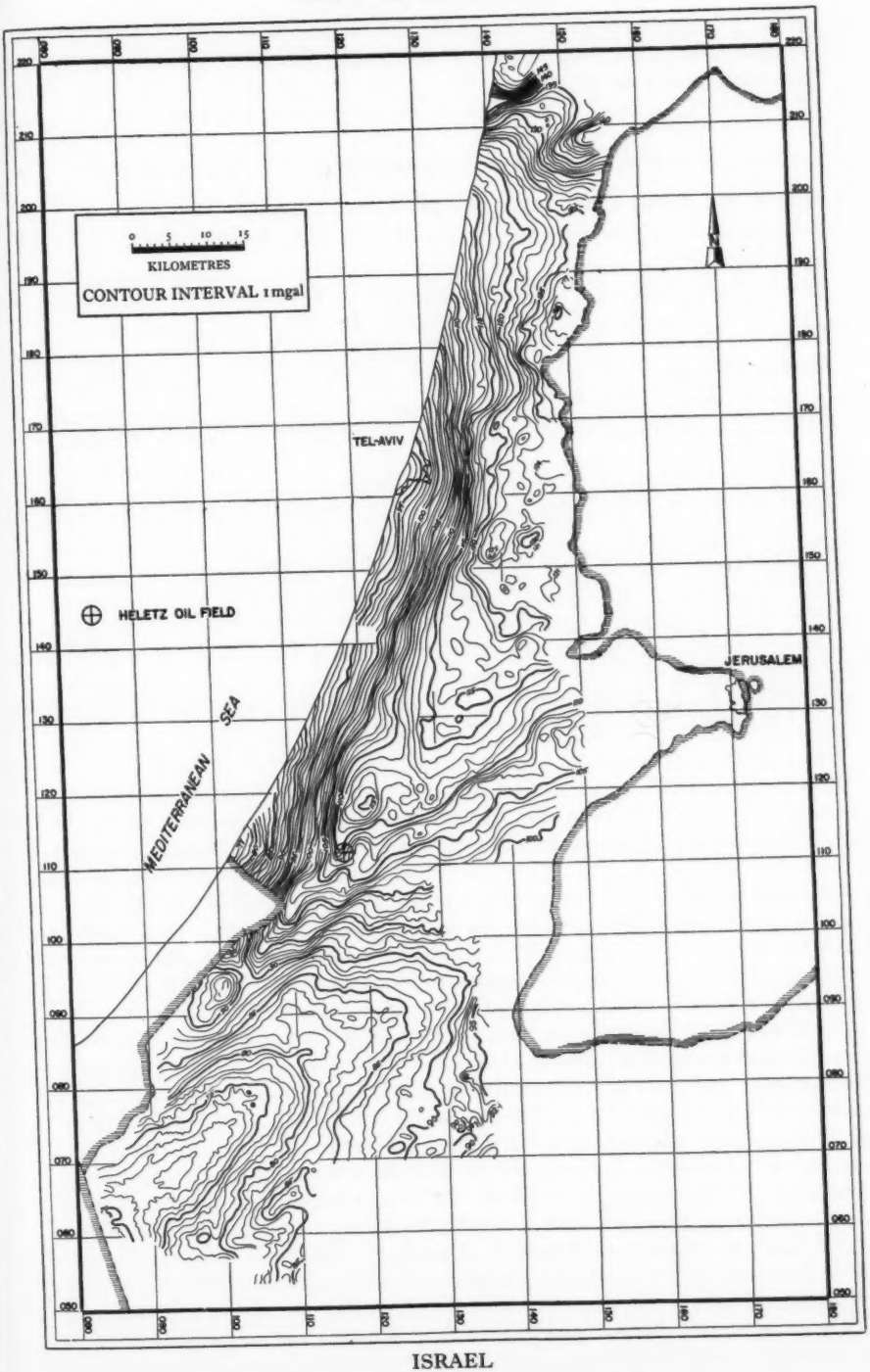
1. that the depth of the oceans be fathomed everywhere, and
2. one can solve the tidal mathematical equations.

Nowadays, the ocean depths are known with tolerable accuracy. The difficulty is in carrying out the mathematical analysis. We are planning to tackle this problem with the aid of WEIZAC. It is our hope to crack this problem long before the 200th anniversary of Laplace's theory in 1975.

As for the atmosphere, I shall only mention here that the late mathematician John von Neumann developed a method of numerical weather forecasting using electronic computers. There has been considerable progress along this line at the hands of Professor J. Charney (1955) of M.I.T. Recently the Meteorological Service in Tel-Aviv made some numerical weather forecasts with the aid of WEIZAC. It is not certain yet whether the wild weather will consent to be tamed by these mathematical tactics. I find thunderstorms particularly a nuisance because WEIZAC is scared of them.

Finally, I wish to bring before you a problem in the upper atmosphere. As in the oceans, there exists also in the atmosphere a barometric tide. Over 20 years ago I produced a theory (Pekeris 1937), according to which one can explain most of the observed features of atmospheric tides, provided one makes the assumption that the temperature increases with height between the levels of 40 to 60 km above ground. At that time 40 km was about the ceiling to which instruments could be sent for atmospheric soundings. After the war it became possible, for the first time, to probe the atmosphere with the aid of rockets to heights of a hundred kilometres and more. These soundings not only verified the above mentioned assumption, but also confirmed another hypothesis which I made, namely that between the levels of 60 and 80 km the temperature *decreases* with increasing height.

I mention this experience for the benefit of our young geophysicists who are busy nowadays making various assumptions about the internal constitution of the



ISRAEL

FIG. 1.—Map of total gravity, Bouguer anomalies.

Earth. It is not inconceivable that in the not remote future it will become possible to probe directly the deep interior of the Earth. Therefore my warning to you: "Scholars, mind your words."

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Geophysical Discussion of the Royal Astronomical Society

A discussion was held in the rooms of the Royal Astronomical Society on 1958 February 21 at 16^h 15^m on **The Transition from the Ionosphere to Interplanetary space** under the Chairmanship of *Mr J. A. Ratcliffe*. He opened the discussion by introducing the speakers.

Professor Sydney Chapman outlined his static theory of the heating of the outer atmosphere by the solar corona. The corona, he said, consisted mainly of hydrogen plus about 10 per cent helium, and had a temperature of the order of 1 million degrees at 0·06 solar radii above the photosphere. Such a gas must have a high thermal conductivity, proportional to $T^{5/2}$, and heat loss would be mainly by conduction. Assuming a static, spherically symmetrical corona, in a steady state, the radial distribution of temperature would give about 0·22 million degrees at the Earth's distance. Thus heat should come in through the F2 layer from this hot solar gas, which merged with the Earth's atmosphere, now known to be many radii greater than the Earth. The variation of temperature with distance from the sun permitted an estimate to be made of how the number density of electrons (or protons) varied with this distance. *Chapman* put this at the order of $10^3/cm^3$ at the Earth's distance; a figure he said agreed with estimates from the study of zodiacal light. These estimates of temperature and electron density would be reduced if the solar magnetic field were great enough to retard thermal conductivity in this ionized gas. A study of eclipse values of the electron density to 20 solar radii suggested that there was no pronounced effect of any such interplanetary magnetic field. The corona was not static and material ejected from the sun must affect the ionosphere as well.

Dr D. E. Blackwell followed with an account of optical observation of the zodiacal light. This light was seen as a cone standing above the eastern horizon before sunrise or above the western horizon after sunset. It was caused by the scattering of sunlight by interplanetary material. Observation of the light permitted estimates to be made of the density of this material as the brightness depended on the total contribution from the scattering elements along a particular line of sight. Thus the density of this material, which was supposed to be dust and free electrons, could be estimated for a tube extending across the Sun's corona. Observations tangential to the Earth's orbit (at an angle of 90° from the Sun) yielded results for the density in the neighbourhood of the Earth. A study of the polarization of the light permitted an estimate to be made of the relative contributions of dust and free electrons as the polarization of the electron component could be calculated and that of the dust was assumed to be zero. A slide was shown which gave these results out to 200 solar radii. It showed a decline from a density of about 5×10^4 electrons/cm^3 near the Sun to a value slightly less than 10^3 electrons/cm^3 at the extreme distance. But the plane of symmetry was close to the ecliptic, suggesting that gravitational forces dominated electromagnetic forces. This argued against an electron component in the scattering material. A study of the polarization led

to two sets of conditions: the electron density decreased slowly with distance from the Sun, and the dust component had a negative polarization, or the electron density decreased sharply at about the distance the Earth was from the Sun. The speaker felt the simplest explanation to be that the electron component did not exist.

Dr J. V. Evans of the Jodrell Bank Observatory, Manchester University, spoke of the results which might be obtained from a consideration of radio echoes reflected from the moon. A plane polarized radio wave was propagated along the Earth's magnetic field as two circularly polarized waves with different phase velocities and senses of rotation; this caused rotation of the plane of polarization. This was called the Faraday effect and might be observed in radio signals passing through the ionosphere either by reflection from the moon or by transmission from an artificial satellite. For radio wave frequencies above 100 Mc/s a quasi-longitudinal propagation held true for ray directions up to 80° to the magnetic field direction. In addition to the frequency, the amount of rotation was dependent on the component of the geomagnetic field along the ray path, the zenith angle, and the electron density. The latter was the function least well known and open to examination by such observations. The models of the ionosphere considered in the light of rocket data suggested that only 3 per cent of the rotation would occur beyond the ionosphere. Moreover, studies of the fading of Moon echoes showed changes in the ionospheric electron content. These were done at Jodrell Bank by observing on two closely spaced frequencies (119·28 Mc/s and 120·72 Mc/s) and permitted determination of the total rotation occurring in the region 0–600 km. It was found that about three times the number of electrons occurred above the F2-region (above about 350 km) as do below it, and that the rotation produced was variable. It would be difficult to measure the small rotation produced in a moon echo by electrons in the region beyond the ionosphere. Possibly a comparison of the total rotation which occurred in a Moon echo with that which occurred in signals transmitted from artificial satellites would overcome this difficulty.

Dr K. Weekes of the Cavendish Laboratory, Cambridge University, then spoke of deductions made about the ionosphere from observations of artificial Earth satellites. Measurement of the Doppler effect, the direction of arrival, and the Faraday rotation for radio waves propagated from a satellite, could all yield information about the ionosphere. So too could observations made with the satellite at different heights, particularly above the F2 peak of ionization, and observations of scintillations of the signals. These effects were closely related and only very accurate measurements could yield independent information. (The Doppler shift should be measured to 0·5 c/s and the angle of arrival to 1 min of arc.) The observations made on satellite 1957 α had not been sufficiently accurate, nor had the orbit been measured with sufficient accuracy to deduce the ion density though the results which were obtained were consistent with the known ionosphere. Scintillations of the signal strength on 40 Mc/s were observed when 1957 α was north of Cambridge and at a height above 280 km. This showed that the irregularities in the F2 region occurred above this height.

The final speaker was the Chairman, *Mr J. A. Ratcliffe*. He summarized recent American work on audio-frequency atmospherics, particularly that of Helliwell at Stanford University, and Gallet at the National Bureau of Standards, Boulder, Colorado. This work followed on from the theory of Storey that an electromagnetic disturbance, caused by a lightning flash, was guided out into the ionosphere and

back, along a line of the geomagnetic field. Dispersion occurred in the ionosphere giving rise to the effect which is characterized in their name, "whistlers". This required an electron density of about $3 \times 10^2/\text{cm}^3$ at a distance of three Earth radii. Three new points had arisen. First, examination of the time-variation of frequency when extended above audio frequencies showed that sometimes maximum dispersion occurred at a particular frequency; second, multiple whistlers were now known, the components of which seemed to have travelled different paths, suggesting a filamentary structure in the outer atmosphere; and third, it had been suggested that a second type of naturally occurring audio-frequency atmospheric existed which was caused by charged particles streaming from the sun and exciting electromagnetic waves by a mechanism similar to that of a travelling-wave-tube amplifier. Thus a model having a striated outer atmosphere was suggested by these recent studies.

In the discussion which followed these papers, *Dr A. Hewish* described observations made at Cambridge of radio stars observed through the solar corona. Scattering of the incident radio waves could be observed up to 20 solar radii from the Sun and was thought to be due to irregularities in the corona and held there by a solar magnetic field extending at least this far. Such a field would reduce the thermal conductivity in the corona. *Dr R. d'E. Atkinson* expressed the view that the picture of a static corona presented by *Chapman* would entail considerable damping of the planets' orbital motions. *Professor Chapman* replied that the corona was certainly more complex than the static picture suggested; it was variable and there was a solar magnetic field; but the point he wished to stress was that this was a very hot gas and the Earth a cool body passing through it.

Professor C. W. Allen called attention to the configuration of the solar magnetic field. At a distance of a few radii from the Sun this would be directed towards the Earth. *Professor V. C. A. Ferraro* agreed that this would not greatly affect the thermal conductivity in Chapman's theory.

The Chairman then commented on the fact that a dilemma seemed to exist between the different estimates of the electron density at distances of 3–4 earth radii from the Earth. He asked *Dr Blackwell* if the estimates made from observations of whistling atmospherics were too high to be in agreement with optical studies. *Professor Chapman* was asked if seasonal variations in the F2-region could arise from the varying distance of the Earth from the Sun. In reply, *Dr Blackwell* said that a density of $10^3/\text{cm}^3$ to heights of 4 earth radii seemed permissible and perhaps $50–100/\text{cm}^3$ at 10–20 radii. There was some optical evidence for a greater extension of the atmosphere than had previously been countenanced. Recent observations of the enhancement of the 5577 O I emission line in zodiacal light by Karimov of Russia supported this. *Professor Chapman* replied that he doubted if the variation in distance of the Sun would have a marked effect on the ionosphere. He thought the density of the corona around the Earth would be very sensitive to temperature and for this reason his theory was not dependent on the zodiacal light measures of electron density.

In reply to a general question about the solar magnetic field, *Professor Allen* said this was about 1 G in the Sun's polar regions. He thought a dynamic theory would have the essential features of Chapman's static theory. Allen said it was not proved that the electron density observed at 3–4 earth radii was true for astronomical space. It might be part of the Earth's atmosphere.

A point was raised about shifts in wavelength of the emission spectrum

from the upper atmosphere. The temperatures postulated by *Chapman* might be expected to cause observable shifts. *Dr R. A. Lyttleton* suggested that the sudden disruption of comet tails was evidence of interplanetary gas.

The Chairman thanked the speakers and other contributors to the discussion and adjourned the meeting at $18^{h}\ 15^{m}$.

Book Review

The Ionosphere

Karl Rawer

(Crosby Lockwood and Son, London), 1958, 42/-

Over the past twenty years, a considerable amount of experimental data on the spatial and temporal changes in ionospheric structure has accumulated. In spite of this, no definitive theory has yet appeared which can account fully for the observed behaviour of the ionized layers. The main barriers to a more satisfactory understanding of the ionosphere are the paucity of the information, not only about the structure and composition of the upper levels of the atmosphere, but also about the ultra-violet and X-radiation, and the streams of atomic particles, emitted by the sun. In the absence of this basic information, ionospheric theory is necessarily based on various plausible but tentative working assumptions, each of which has its supporters, which often permit partial interpretations of the observational facts.

The author of a book on the ionosphere will almost certainly have his own opinions as to which interpretations are most likely to be the true ones and, in a short text, he may decide to concentrate on these when discussing ionospheric behaviour. It seems preferable, however, that he should try to introduce and comment on several alternative interpretations leaving the enquiring reader to follow up the original papers on the subject; this is the course which has been adopted by Dr Rawer.

The first half of the book deals with the basic physics of the ionization of the Earth's atmosphere by solar radiation, and the refraction and absorption of radio waves in an ionized gas; it also contains brief references to the experimental techniques used to "sound" the ionosphere. Short sections are included which outline the relevant facts about geomagnetism and solar ionizing radiation. In the second half, descriptions are given of the regular behaviour and the abnormal perturbations of the ionospheric layers as actually observed. In addition, there are important sections which deal with the propagation of radio waves in the spherical shell between the ionosphere and the Earth's surface, and with the related problems of ionospheric forecasting.

At the end of the IGY, many individuals who are now engaged in making ionospheric observations of all kinds will, no doubt, turn to the equally important task of studying and interpreting the unprecedented collections of information which are now being assembled in the World Data Centres. For these, Dr Rawer's book will provide not only an excellent bird's eye view of the whole subject but also, in the references, the keys to the literature in many of the specialized fields of research.

The publication of the English translation has given the author the opportunity for making some small additions and amendments to the earlier German text

and for a considerable expansion of the list of references which now includes entries up to 1955. On the whole the translation has been well done, but readers whose first language is not English may occasionally hesitate when confronted with words like "doubtlessly" and "altitudinal" and by American colloquialisms such as "gotten into . . . orbit". Of the few typographical errors, the only one of any moment noticed by the reviewer is the substitution of x/dx for dx near the top of p. 157. A useful feature of the well-chosen plates and diagrams is that three-quarters of them are based on actual experimental data.

C. M. Minnis

РЕЗЮМЕДОКЛАДОВ, В ПЕРЕВОДЕ НА РУССКИЙ ЯЗЫК

О ИНТЕРПРЕТАЦИИ *Pd*

Гарольд Джеффрис

Разбираются наблюдения Лейман широкой фазы вблизи 21°, отстающей на несколько секунд от первого появления *Pr*, при помощи применения двух крайних гипотез. По первой гипотезе — расстояние изменяется как квадратичная функция sin *e* таким образом, что $d\Delta/de$ исчезает при 21°, где находится точка заострения на кривой времени. Найдено, что величины $dt/d\Delta$ для *Pr* и *Pd* таковы, что нет возможности найти место для более близкой точки заострения. По второй — кубическая форма *t* принимается для *Pd* до 21°. *Pr* должно появиться между 17° и 18°. Луч *Pd*, появляющийся при 21°, должен достигнуть приблизительно 0·03 *R* ниже разрывности Мохоровичича. Периоды *Pr* приводят к заключению, что при переходе должно быть значительное нарушение непрерывности градиента скорости, но всего лишь слабое нарушение непрерывности скорости. Более близкая точка заострения на кривой будет находиться около 15°, но по всей вероятности ее нельзя будет обнаружить, т.к. она появляется менее секунды после первого прибытия.

ДИНАМИЧЕСКОЕ ВЫЧИСЛЕНИЕ СИСТЕМЫ ЭКВАТОРИАЛЬНЫХ ТЕЧЕНИЙ В ТИХОМ ОКЕАНЕ, В ЧАСТНОСТИ В ПРИЛОЖЕНИИ К ЭКВАТОРИАЛЬНОМУ ВНУТРЕННЕМУ ТЕУЕНИЮ

Коджи Хидака и Ютака Нагата

Для исследования течений вблизи экватора используются решения динамических уравнений установившегося состояния, заключающих в себе выражения силы Кориолиса, градиента давления, а также горизонтального и вертикального смещений. Коэффициенты горизонтального и вертикального смещений принимаются соответственно в 10^8 и 10^2 c.g.s. независимо от их приложения. Распределения среднего напряжения ветра и среднего давления вычисляются на основании наблюдаемых данных. Результаты указывают на течение в восточном направлении, под поверхностью вблизи экватора, что, повидимому, объясняет экваториальное внутреннее течение.

ОТЧЕТ РАБОТЫ С СЕЙСМОГРАФАМИ ПРОДОЛЖИТЕЛЬНЫХ ПЕРИОДОВ

Х. Бениофф и Ф. Пресс

Описываются системы сейсмографов для продолжительных периодов, работающих в Пасадене. Среди полученных результатов даются: продолжение кривых групповой скорости волн Ралея и волн G оболочки, выделение этих волн из землетрясений в диапазоне величины $5{,}75 \div 7$ и регистрация необычных волн массы с непредвиденными компонентами длительных периодов.

МАГНИТНОЕ ПОЛЕ И ЦЕНТРАЛЬНОЕ ЯДРО ЗЕМЛИ

Леонид Кнопов и Гордон Дж. Ф. Макдональд

Обсуждается гипотеза жидкого состояния внутреннего ядра Земли и проницаемости его сильным магнитным полем. Доказывается, что жидкое внутренее ядро с сильным магнитным полем не должно передавать волн сдвига, что противоречит предположению основанному на магнитогидродинамической жёсткости внутреннего ядра. В пользу гипотезы о сильном магнитном поле в жидком внутреннем ядре говорит отсутствие фазы *PKJKP* в сейсмографических измерениях. Если скачок скорости волны *P* на границе внутреннего ядра обусловлен магнитным полем, то напряженность поля должна быть порядка $5 \cdot 10^6$ гаусс, не завися от того, является ли поле тороидальным или неправительной геометрической формы. Сильное тороидальное поле вызывает кажущуюся эллиптичность внутреннего ядра. Такая эллиптичность не наблюдается, следовательно, если поле во внутреннем ядре тороидально, его напряженность должна быть менее $5 \cdot 10^5$ гаусс. Поле неправильной геометрической формы не вызывает явления эллиптичности и нельзя определить предела величине такого поля.

Энергия, необходимая для магнитогидродинамической жёсткости внутреннего ядра, слишком велика. Магнитная энергия делится поровну между внутренними и внешним ядрами. Гипотеза сильного магнитного поля во внутреннем ядре не может объяснить наблюдаемых изменений коротких периодов поверхностного поля.

ПАЛЕОГЕОГРАФИЧЕСКАЯ КАРТА ВОСПРОИЗВЕДЕННАЯ ПРИ ПОМОЩИ ПАЛЕОМАГНЕТИЗМА

Э. Ирвинг

Описывается метод для воссоздания карт распределения континентов в прошлом, используя палеомагнетические измерения. Метод поясняется пользуясь результатами, полученными в Европе, Австралии, Индии, Африке и Южной Америке, с целью воспроизведения по взможности прошлого рельефа этих континентов.

КАУСТИКА ОБРАЗОВАННАЯ ВОЛНАМИ ПРОХОДЯЩИМИ ЧЕРЕЗ ЯДРО ЗЕМЛИ

Б. Гутенберг

Исследуется каустика образованная фазами SPK, PKS, PKP и др., проходящими через ядро земли. Вне 5° от каустики не было найдено импульсов преломленных волн, периоды которых 2 секунды и более. У коротких волн, опережающих главную фазу PKIKP на расстояние от 125° до 240°, самая глубокая точка вероятно внутри или вблизи переходной зоны, между жидким внешним и вероятно твердым внутренним слоями ядра, и мало вероятности, что они имеют отношение к каустике PKP около 145°, хотя кривые их передвижения оканчиваются вблизи этой каустики. Наблюдаемое действие волн длинных периодов, преломленных вне каустик нескольких фаз, меньше максимального действия рассчитанного Джеффрейсом.

Geophysical Journal

RECOMMENDATIONS TO AUTHORS

1. Authors are advised to follow the general suggestions for the preparation of scientific papers given in *Notes on the preparation of papers to be communicated to the Royal Society* and in *The printing of mathematics* by T. W. Chaundy, P. R. Barrett and C. Batey (Oxford University Press, 1954). In particular the abbreviations for the names of periodicals used in the *Geophysical Journal* are those given in *Notes on the preparation of papers* . . . and the symbols, signs and abbreviations are those recommended for British scientific publications by the Symbols Committee of the Royal Society.

2. Papers should be typewritten with double spacing and on one side of the paper only. Two copies, one of them the top copy, should be submitted. All papers must be accompanied by a summary; summaries are not required for *Letters to the Editors*.

3. Tables are printed without rules. They should be numbered serially with Arabic numerals. Table headings should be brief. Units should be placed at the head of the column. Tables should be typed on separate sheets and their positions in the text indicated on the copy.

4. Illustrations, especially photographs, should be kept to a minimum, and the same information should not in general be given in both tables and illustrations. Line drawings should be in dense black ink on smooth white board or transparent tracing film. Original drawings must not be lettered; lettering should be indicated on copies or photoprints which must be supplied in addition to the originals. Wherever possible, lettering should be kept outside the diagram so that it may be set in type. Diagrams should be drawn at twice to three times the size at which they will be printed. The maximum dimensions for diagrams, when reduced and with their lettering and captions, are 8 in by 5 in. A wide variety of flat tints of dots, lines and shadings can be applied to line drawings by the blockmaker. Photographs for reproduction should be unmounted glossy prints and should be accompanied by lettered prints.

5. References should be quoted in the form recommended by the Royal Society but authors may also quote titles of papers if they wish. References in the text are made in the form (Smith 1940).

6. Authors whose work involves much mathematics should follow carefully the recommendations in *The Printing of Mathematics*.

7. The Editors will be glad to advise authors on any special points of difficulty arising in the preparation of papers for the *Journal*.

Contents

PAGE

HAROLD JEFFREYS, On the interpretation of *Pd* 191

KOJI HIDAKA and YUTAKA NAGATA, Dynamical computation of the equatorial current system of the Pacific, with special application to the equatorial undercurrent 198

HUGO BENIOFF and FRANK PRESS, Progress report on long period seismographs 208

LEON KNOPOFF and GORDON J. F. MACDONALD, The magnetic field and the central core of the Earth 216

E. IRVING, Palaeogeographic reconstruction from palaeomagnetism 224

B. GUTENBERG, Caustics produced by waves through the Earth's core 238

Notes on Progress in Geophysics:

A. L. HALES, Geophysics in South Africa 249

C. L. PEKERIS, Geophysics, Pure and Applied 257

Geophysical Discussion of the Royal Astronomical Society, 1958 February 21

The transition from the ionosphere to interplanetary space 263

Book review: The Ionosphere 267

Russian translations of summaries of original papers 269

РЕЗЮМЕ ДОКЛАДОВ, В ПЕРЕВОДЕ НА РУССКИЙ ЯЗЫК 269

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